STAT 582 Homework 2

Due date: In class on Monday, February 13, 2006

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5. Let X_n have normal distribution with mean 0 and variance σ_n^2.

   (a) Assume that $\{X_n\}_n$ is u.i. Show that the variances must be uniformly bounded, i.e., there exists K such that $\sigma_n \leq K$ for all n.

   (b) Assume that the variances are uniformly bounded, i.e., assume there exists K such that $\sigma_n \leq K$ for all n. Show that for all n

   $$\int_{|X_n|>a} |X_n| dP \leq \frac{2}{\sqrt{2\pi}} K \int_{a/K}^{\infty} y \exp(-\frac{y^2}{2}) dy$$

   and conclude that $\{X_n\}_n$ is u.i.

   In summary, you showed that a sequence of zero mean normal variables is u.i. iff their variances are unformly bounded.

6. Suppose $\{X_n\}_n$ and $\{Y_n\}_n$ are two u.i. families defined on the same probability space. Is $\{X_n + Y_n\}_n$ u.i.? Show your argument.

   Hint: Triangular inequality $|X_n + Y_n| \leq |X_n| + |Y_n|$ a.s.

7. Let $\{X_n\}_n$ be a sequence of i.i.d. random variables with mean zero and variance σ^2. Let $\{a_n\}_n$ be a sequence of real numbers. Set

   $$S_n = \sum_{i=1}^{n} a_i X_i.$$

   Prove: $\{S_n\}_n$ converges in $L_2 \iff s_n := \sum_{i=1}^{n} a_i^2$ converges in $\mathbb{R}$.

   Hint: you can not use the limiting random variable $\sum_{i=1}^{\infty} a_i X_i$ to show convergence before you have not established that the limit actually exists. So, use the C... criterium.

8. Let $\{Y_n\}_n$ be a sequence of independent Gaussian random variables with mean zero and variance σ_n^2. Set $S_n = \sum_{i=1}^{n} a_i X_i$. Under what assumptions on the sequence of variances does S_n converge in L_2?