ELEC 533 Homework 10

Due date: In class on Friday, December 1st, 2000

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32. Recall that the power spectral density $S_X(\nu)$ of a wss process X_t is defined as the Fourier transform of its auto-correlation function $R_X(\tau)$. Show the following relations:

    (a) If $Y_t = aX_t$ then $R_Y(\tau) = |a|^2 R_X(\tau)$ and $S_Y(\nu) = |a|^2 S_X(\nu)$.

    (b) Let U_t and V_t be orthogonal processes, meaning that $\mathbb{E}[U_t V_s] = 0$ for all t and s. Set $W_t = U_t + V_t$. Show that $R_W(\tau) = R_U(\tau) + R_V(\tau)$ and conclude that $S_W(\nu) = S_U(\tau) + S_V(\tau)$.

    (c) Let X_t be a zero mean process, meaning that $\mu_X(t) = 0$. Set $Z_t = X_t + b$. Show that $R_Z(\tau) = R_X(\tau) + |b|^2$ and $S_Z(\nu) = S_X(\nu) + 2\pi|b|^2\delta(\nu)$.

    (d) If A_t is a wss process such that $R_A(\tau) = -R_X''(\tau)$ then $S_A(\nu) = \nu^2 S_X(\nu)$. (Here, R'' denotes the second derivative.)

    Most of these questions can be derived from properties of the Fourier transform.

33. Compute the spectral density for processes with the following auto-correlation:

    (a) $R_X(\tau) = \exp(-\beta|\tau|)$.

    (b) $R_X(\tau) = \max(1 - |\tau|/T, 0)$. HINT: R_X is the convolution of two extremely simple functions with extremely simple Fourier transforms.

    The first example is the auto-correlation of a Gauss-Markov process, the second can be obtained running white noise through an integral averager (see class). Indeed: both auto-correlations decay very fast, indicating that the processes are 'short range dependent'.