ELEC 533 Homework 7

Due date: In class on Wednesday, November 1st, 2000

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25. There is a theorem which states that if there is a random variable Y with $E[|Y|^2] < \infty$ such that $|X_m| \leq Y$, almost surely, and if X_m converges in probability then X_m converges also in the mean square sense. Your task is to prove a simplified version of this fact which goes as follows:
Suppose that
$$X_m \xrightarrow{\text{i.p.}} X$$
and suppose that there is a constant c such that $|X_m| \leq c$ for all n, then
$$X_m \xrightarrow{\text{m.s.}} X.$$

26. Suppose that
$$X_m \xrightarrow{\text{D}} X$$
and that there is a constant c such that $P[X = c] = 1$. Show that
$$X_m \xrightarrow{\text{i.p.}} X$$

27. Let $\Omega = [0, 1]$ be sample space (universal set), where the event class (σ-algebra) are all Borel subsets of Ω. The probability distribution on Ω is uniform. Define a sequence of random variables X_m from Ω to $\mathbb{R}$ as follows: Write m uniquely in the form $m = 2^n + k$ with $0 \leq k \leq 2^n - 1$ and set
$$X_m = X_{2^n+k}(\omega) = \begin{cases} 1 & \text{for } \omega \text{ in } [k2^{-n}, (k+1)2^{-n}], \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show that X_m converges in probability but not almost surely.

   (b) Find a subsequence X_{m_l} so that
   $$X_{m_l} \xrightarrow{\text{a.s.}} 0$$

   (c) Find a random variable Y so that
   $$|X_m| \leq Y,$$
   and
   $$E[|Y|^2] < \infty$$

   (d) A theorem states that if a sequence converges in probability (as our sequence, see 27a) and is mean square dominated (as our sequence, see 27c) then that sequence also converges in the mean square sense. For our sequence, this means
   $$X_m \xrightarrow{\text{m.s.}} 0.$$
   Verify this for our sequence X_m by showing that
   $$E[|X_m|^2] \longrightarrow 0$$

   (e) Define
   $$Y_{2^n+k} = 2^n X_{2^n+k}$$
   Show that
   $$Y_m \xrightarrow{\text{i.p.}} 0.$$
   Also show that Y_m does not converge in the m.s..