ELEC 533 Homework 1

Due date: In class on Wednesday, September 10th, 2003

Instructor: Dr. Rudolf Riedi

1. Using the three axioms of probability, prove the following properties of probability.
   (a) If $A \subset B$ then $P[A] = P[B] - P[A^c \cap B]$.
   (b) If $A \subset B$ then $P[A] \leq P[B]$. (Use problem (1a).)

2. Verify that the following are valid probability laws. To this end determine whether the underlying probability space is discrete or continuous and use the criteria established in class.
   (a) The binomial law $(0 \leq \theta \leq 1)$ on $\Omega = \{0, 1, ..., n\}$
   $$P[\{a\}] = \frac{n!}{(n-a)!\ a!}\ \theta^a\ (1-\theta)^{n-a}. \tag{1}$$
   (b) The Poisson law $(\lambda > 0)$ on $\Omega = \mathbb{N}_0 = \{0, 1, 2, \ldots\}$
   $$P[\{a\}] = \frac{e^{-\lambda}}{a!}\ \lambda^a. \tag{2}$$
   (c) The exponential law $(\lambda > 0)$ on $\Omega = \mathbb{R}$
   $$P[(-\infty, y]] = \begin{cases} \int_0^y \lambda\ e^{-\lambda\tau} d\tau & \text{if } y \geq 0 \\ 0 & \text{if } y < 0. \end{cases} \tag{3}$$
   (d) The uniform law on $[a, b]$
   $$P[[x, y]] = \frac{y - x}{b - a} \qquad b \geq y \geq x \geq a. \tag{4}$$
   (e) The following law on $\mathbb{N} = \{1, 2, \ldots\}$
   $$P[\{n\}] = 2^{-n}. \tag{5}$$

3. Let $\Omega_4 = \{1, 2, 3, 4\}$.
   (a) Find the smallest σ-algebra containing $A = \{1, 3\}$.
   (b) Find the smallest σ-algebra containing $A = \{1\}$, $B = \{3\}$.

4. Two events A and B are called *independent* if $P[A \cap B] = P[A] \cdot P[B]$. Prove that if A and B are independent events, then A and B^c are also independent.

5. In the following, some number of dice are thrown simultaneously. A "straight" is the event when all numbers from 1 through 6 are observed.
   (a) What is the probability that a "straight" occurs when six dice are thrown simultaneously?
   (b) What is the probability that a "straight" occurs when seven dice are thrown simultaneously?