ELEC 533 Homework 8

Due date: In class on Friday, November 21, 2003

Instructor: Dr. Rudolf Riedi

30. Let $\{N_t, t \in \mathbb{R}\}$ be a Poisson process with intensity λ. This means that $\{N_t, t \in \mathbb{R}\}$ is a renewal process, such that for any fixed t the r.v. N_t is a Poisson r.v. with mean λt, the number of arrivals over disjoint intervals are independent, and the interarrival times X_k between events are independent exponential r.v. with parameter λ.

    (a) Compute the second order joint statistics $F_{N_t N_s}$, i.e. compute $P[N_t = n,\, N_s = m]$.
    Hint: Increments.

    (b) Poisson processes are constructed pathwise, so their f.d.d. are automatically consistent. For exercise, verify the consistency for the second order statistics, i.e. verify that

    $$\sum_{m=0}^{\infty} P[N_t = n,\, N_s = m] = P[N_t = n].$$

31. Suppose X_i ($i \in \mathbb{N}$) is a sequence of iid random variables and suppose

    $$P[X_i = 1] = P[X_i = -1] = 1/2$$

    for all i. Define a random process $\{Y_t, t \in \mathbb{R}\}$ by

    $$Y_t = X_i \qquad \text{for all } t \text{ such that } i - 1 < t \leq i.$$

    (a) Sketch a typical sample path of $\{Y_t, t \in \mathbb{R}\}$.

    (b) Find the mean and autocorrelation function of $\{Y_t, t \in \mathbb{R}\}$. Is the process wide sense stationary (wss)? Show your argument.

    (c) (voluntary exercise; not required) Suppose A is a r.v. independent of the X_i and uniformly distributed in $[0, 1]$: $A \sim \mathcal{U}([0, 1])$. Define the random process $\{Y_t, t \in \mathbb{R}\}$ by

    $$Z_t = Y_{t+A}$$

    Sketch a typical sample path of $\{Z_t, t \in \mathbb{R}\}$.

    (d) Is Z_t wss? Is Z_t second order stationary? Why or why not?

32. Suppose A and B are independent Gaussian random variables, i.e. $A \sim \mathcal{N}(0, 1)$ and $B \sim \mathcal{N}(0, 1)$. Define the random process $\{X_t, t \in \mathbb{R}\}$ by

    $$X_t = A + Bt + t^2.$$

    (a) Compute the following marginal distributions: ϕ_{X_t} for arbitrary t and ϕ_{X_1, X_5}.

    (b) Are $X_1, X_2, \ldots, X_5$ jointly Gaussian?

    (c) Find $\mathbb{E}[X_5 \,|\, X_0]$. This is a prediction of X_5 knowing X_0, and it is a random variable. Compute its variance.

    (d) Find $\mathbb{E}[X_5 \,|\, X_0,\, X_1]$. This is a prediction of X_5 knowing X_0 and X_1, and it is also a random variable. Compute its variance.