ELEC 533 Homework 1

Due date: In class on Tuesday, September 14th

Instructor: Dr. Rudolf Riedi

1. Using the axioms of probability, prove the following properties of probability.
   - If $A_1, A_2, \cdots \in \mathcal{A}$, then
   $$P[\bigcup_i A_i] \leq \sum_{i=1}^{\infty} P[A_i]$$
   .
   - If $A_1, A_2, \cdots \in \mathcal{A}$, and $A_i \subseteq A_{i+1}$, then
   $$P[\bigcup_i A_i] = \lim_{n\to\infty} P[A_n] = \sup_n P[A_n]$$
   .
   - If $A_1, A_2, \cdots \in \mathcal{A}$, and $A_i \supseteq A_{i+1}$, then
   $$P[\bigcap_i A_i] = \lim_{n\to\infty} P[A_n] = \inf_n P[A_n]$$
   - If $P[B] > 0$, and $A_i \bigcap A_j = \emptyset$ for $i \neq j$, then
   $$P[\bigcup_i A_i \mid B] = \sum_{i=1}^{\infty} P[A_i \mid B]$$
2. Show that the following are valid probability laws.
   - A binomial law ($0 \leq \theta \leq 1$)
   $$P[\{a\}] \quad = \quad \begin{cases} \frac{n!}{(n-a)!\ a!}\ \theta^a\ (1-\theta)^{n-a}; & a \ = \ 0,1,2,\cdots,n \\ 0 & \text{otherwise} \end{cases}$$
   - A Poisson law ($\lambda \geq 0$)
   $$P[\{a\}] \quad = \quad \begin{cases} \frac{e^{-\lambda}}{a!}\ \lambda^a; & a \ = \ 0,1,2,\cdots \\ 0 & \text{otherwise} \end{cases}$$
   - An exponential law ($\lambda \geq 0$)
   $$P[[x,y)] \quad = \quad \begin{cases} \int_x^y \lambda\ e^{-\lambda\tau} d\tau & y \geq x \geq 0 \\ 0 & 0 \geq y \geq x \end{cases}$$
3. Let $\Omega_6 = \{1,2,3,4,5,6\}$.
   - Find the smallest σ-algebra containing $A = \{1,3,5\}$.
   - Find the smallest σ-algebra containing $A = \{1\}$, $B = \{3\}$ and $C = \{5\}$.
   - Find the smallest σ-algebra containing $A = \{1\}$, $B = \{1,3\}$ and $C = \{1,3,5\}$.
4.
   - Prove the law of total probility and the Bayes rule.
   - Prove that if A and B are independent events, so are A and B^c.
5. A "straight" is the event when all numbers from 1 through 6 are observed when several dice are thrown simultaneously.
   - What is the probability that a "straight" occurs when six dice are thrown simultaneously?
   - What is the probability that a "straight" occurs when seven dice are thrown simultaneously?