ELEC 533 Homework 10

Due date: In class on Thursday, December 2nd

Instructor: Dr. Rudolf Riedi

33. Recall that the power spectral density $S_X(\nu)$ of a wss process is defined as the Fourier transform of its auto-correlation function $R_X(\tau)$. Show the following relations:

    (a) If $Y_t = aX_t$ then $R_Y(\tau) = |a|^2 R_X(\tau)$ and $S_Y(\nu) = |a|^2 S_X(\nu)$.

    (b) If $W_t = U_t + V_t$ and U_t and V_t are orthogonal then $R_W(\tau) = R_U(\tau) + R_V(\tau)$ and $S_W(\nu) = S_U(\tau) + S_V(\tau)$.

    (c) If $Z_t = X_t + b$ and $\mu_X(t) = 0$ then $R_Z(\tau) = R_X(\tau) + |b|^2$ and $S_Z(\tau) = S_X(\nu) + 2\pi|b|^2\delta(\nu)$.

    (d) If A_t is a wss process with $R_A(\tau) = -R''_X(\tau)$ then $S_A(\nu) = \nu^2 S_X(\nu)$.

34. Compute the spectral density for processes with the following auto-correlation:

    (a) $R_X(\tau) = \exp(-\beta|\tau|)$.

    (b) $R_X(\tau) = \max(1 - |\tau|/T, 0)$. HINT: R_X is the convolution of two extremely simple functions.

    The first example is the auto-correlation of a Gauss-Markov process, the second can be obtained running white noise through an integral averager (see class).