ELEC 533 Homework 2

Due date: In class on Tuesday, September 21st

Instructor: Dr. Rudolf Riedi

6. Compute the distribution functions, expectations and variances of the random variables with the following densities:

   (a) $X \simeq \mathcal{U}([0, 2\pi])$: Uniform on $[0, 2\pi]$

   $$f_X(x) = \begin{cases} \frac{1}{2\pi} & \text{for } x \text{ in } [0, 2\pi], \\ 0 & \text{otherwise.} \end{cases}$$

   (b) $X \simeq$ Cauchy:

   $$f_X(x) = \frac{1}{\pi(1 + x^2)} \qquad \text{for every } x \text{ in } \mathbb{R}.$$

7. Compute the expectations and variances of the random variables

   (a) $N \simeq \text{Poiss}(\lambda)$: Poisson with parameter $\lambda > 0$, which is given by

   $$P[N = n] = P_N(n) = \begin{cases} e^{-\lambda} \frac{\lambda^n}{n!} & \text{for } n = 0, 1, 2, \cdots \\ 0 & \text{otherwise.} \end{cases}$$

   (b) $X \simeq \mathcal{N}(\mu, \sigma^2)$: Gaussian or normal distribution with parameters $\sigma^2 > 0$ and $\mu \in \mathbb{R}$, which is given through the density

   $$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right).$$

   (You don't have to show that this is indeed a probability density.)

   (c) $X \simeq \exp(\lambda)$: One sided exponential with parameter $\lambda > 0$, which is given through the density

   $$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

8. Let $\Omega = \{1, 2, ..., 6\}$, and $\mathcal{A}_1 = \sigma(\{1, 3, 5\})$ and $\mathcal{A}_2 = \sigma(\{1\}, \{3\}, \{5\})$. These σ-algebras have been computed in HW1, problem 3. Let X be defined as follows:

   $$X(\omega) = \begin{cases} 10 & \text{if } \omega = 1, \\ 10 & \text{if } \omega = 3, \\ 500 & \text{if } \omega = 5, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show that X is a random variable (measurable) over $\mathcal{A}_2$, but not over $\mathcal{A}_1$.

   (b) Compute $\sigma(X)$.

   (c) Change X slightly to obtain a new random variable Y such that $\mathcal{A}_1 = \sigma(Y)$. Similarly, find a third r.v. Z with $\mathcal{A}_2 = \sigma(Z)$.

   (d) Someone is throwing a dice and is telling you for every event in $\mathcal{A}_1$ whether it occured or not. Are you able to compute the value of X? What about Y and Z? How are the answers related to the measurability of X, Y, and Z over $\mathcal{A}_1$?

   (e) Someone is throwing a dice and is telling you the value of X. Are you able to say for each event from $\mathcal{A}_1$ whether it occured or not? Are you able to do so for $\mathcal{A}_2$, or for $\sigma(X)$? Can you determine the value of Y, or Z?

   In (d) and (e) only short answers and no details are expected.