ELEC 533 Homework 9

Due date: In class on Tuesday, November 16th

Instructor: Dr. Rudolf Riedi

30. Suppose A and B are independent Gaussian random variables, i.e. $A \sim \mathcal{N}(0,1)$ and $B \sim \mathcal{N}(0,1)$. Define the random process $\{X_t, t \in \mathbb{R}\}$ by

$$X_t = A + Bt + t^2.$$

   (a) Compute the marginal distributions, i.e. f_{X_t} for fixed, but arbitrary t.

   (b) Find $\mathbb{E}[X_5 \,|\, X_0]$. This is a prediction of X_5 knowing X_0, and it is a random variable. Compute its variance.

   (c) Find $\mathbb{E}[X_5 \,|\, X_0,\, X_1]$. This is a prediction of X_5 knowing X_0 and X_1, and it is also a random variable. Compute its variance.

   (d) What is the joint distribution of X_5 and X_1?

31. Suppose X_i ($i \in \mathbb{N}$) is a sequence of iid random variables and suppose

$$P[X_i = 1] = P[X_i = -1] = 1/2$$

for all i. Define a random process $\{Y_t, t \in \mathbb{R}\}$ by

$$Y_t = X_i \qquad \text{for all } t \text{ such that } i - 1 < t \leq i.$$

   (a) Sketch a typical sample path of $\{Y_t, t \in \mathbb{R}\}$.

   (b) Find the mean and autocorrelation function of $\{Y_t, t \in \mathbb{R}\}$. Is the process wide sense stationary (wss)? Show your argument.

   (c) Suppose A is a r.v. independent of the X_i and uniformly distributed in $[0,1]$: $A \sim \mathcal{U}([0,1])$. Define the random process $\{Y_t, t \in \mathbb{R}\}$ by

$$Z_t = Y_{t+A}$$

   Sketch a typical sample path of $\{Z_t, t \in \mathbb{R}\}$.

   (d) Is Z_t wss? Is Z_t second order stationary? Why or why not?

32. Let $\{N_t, t \in \mathbb{R}\}$ be a Poisson process with intensity λ. This means that $\{N_t, t \in \mathbb{R}\}$ is a renewal process, such that for any fixed t the r.v. N_t is a Poisson r.v. with mean λt, the number of arrivals over disjoint intervals are independent, and the times X_k between arrivals are independent exponental r.v. of parameter λ.

   (a) Compute the autocorrelation $R_N(t,s) = \mathbb{E}[N_t N_s]$.

   (b) Compute the second order joint statistics $F_{N_t N_s}$, i.e. compute $P[N_t = n,\, N_s = m]$.

   (c) Check the consistency for the second order statistics, i.e. show that

$$\sum_{m=0}^{\infty} P[N_t = n,\, N_s = m] = P[N_t = n].$$

   (d) Verify the CLT for Renewal processes in the case of Poisson processes, i.e. show that

$$\frac{N_t - t/\mathbb{E}[X_1]}{\sqrt{t \operatorname{Var}(X_1)/\mathbb{E}[X_1]^3}}$$

   converges to $\mathcal{N}(0,1)$ in distribution as $t \to \infty$. HINT: Use the charateristic function.