Nonparametric Regression for Geographic Visualization and Analysis of Environmental Policy

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Abstract

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The U.S. Department of Agriculture administers and analyzes a large number of surveys of both economic and environmental data. The data collected have traditionally been analyzed on the basis of large geographic regions, and results presented in tables. Through the use of the averaged shifted histogram (ASH) approach to nonparametric regression, it is now possible to analyze survey data on the basis of local areas. The results can be presented as surfaces, or maps where the data are geo-referenced. The spatial analysis of covariates using the ASH has provided new insights into policy applications such as the relation of government payments to agricultural land values, spatial distribution of government payments, and use of agricultural nutrients. In a related use of the ASH estimator, we use estimates of surfaces from survey data to link economic and physical models for analysis of environmental issues. Much economic and environmental data are georeferenced to points which are drawn from a spatial distribution. With the ASH estimator data sets which are georeferenced to different sets of points can be linked for analysis. ASH estimates of surfaces representing physical variables allow analysis that links firm-level economic decisions to spatially distributed processes in the physical environment. The economic and physical effects of alternative policies can then be modeled and the results visualized with nonparametric regression. The ASH is very fast compared to most smoothers, and can be modified to account for a complex survey design in estimation. In this paper, the basic ASH methodology is described and several case studies presented.

1 Introduction

This paper presents the results of application of economic and statistical methods to the study of environmental problems. In these applications, the averaged shifted histogram (ASH) approach to density estimation and nonparametric regression was used to visualize the results from economic models, explore the multivariate relationships, and link economic and physical environmental models. In these studies, we faced large data sets (more than 500,000 observations), as well as data from complex stratified surveys.

The application of smoothing techniques to spatial data has attracted some criticism, on the basis that "real" detail is lost. In an application where data points were very densely and accurately sampled throughout the study region (with digital elevation models, for example), this argument would have some strength. However, survey data by its nature is gathered at widely dispersed or imprecisely located geographic points. The usual practice is to assign the value of each sample point to the area from which the observation was taken. This practice, which results in choropleth maps, is commonly applied to census divisions, county, and state level data. Brillinger (1990) deplores the use of maps which use a constant value for the whole geographic division. He argues that smoothed data provide a more accurate model of the underlying spatial process, and suggests applying a distance dependent local weighting scheme. In a similar spirit, Cowling, et al. (1996) construct nonparametric regression using a nearest-neighbors algorithm to geographically smooth Australian farm survey data. In a study comparing the more traditional method of analysis of spatial data, kriging, with smoothing techniques, Altman (1997) found comparable performance of the methods. In our experience, even if the data are oversmoothed, large-scale geographic relationships in the data are easily identified. In many applications, such as policy analysis at a national level, a simple interpretation without a high level of unnecessary detail is desirable.

In section 2, we describe the ASH density and regression estimation algorithms. Three real current applications of the ASH are described in section 3: the use of the ASH to explore the determinants of crop-land values; a description of an economic model and the application of the ASH density estimator to results from that model; and, finally, the use of the ASH to link data sets and models in an analysis of environmental policy concerning non-point source pollution. Most of our results computed via the ASH are visualized using powerful geographic information system (GIS) software.

2 The Averaged Shifted Histogram

In the ASH algorithm, we operate on binned data. In the simplest application, we are dealing with (x, y, z) data, where (x, y) represents the center of one of our bivariate bins containing one or more primary sampling units. The variables (x, y) are usually some geographic projection of longitude and latitude, respectively. The variable z represents the quantity of interest which has been observed at a known geographic location. For example, z might measure crop land value or nutrient application dollars in farm income. We seek to estimate $\mathrm{E}\left[Z(x, y)\right]$ or $\bar{z}(x, y)$ in areas where the density of observations, $f(x, y)$, is greater than zero.

Let K be a symmetric kernel function with support on $(-1, 1)$ satisfying $\int_{-1}^{1} K(t)\, dt = 1$. Given a positive smoothing parameter h, define the scaled kernel function by $K_h(t) = h^{-1} K\left(h^{-1} t\right)$. The trivariate product kernel density estimator (with possibly different smoothing parameters for each dimension) is given by

$$\hat{f}(x, y, z) = \frac{1}{n} \sum_{i=1}^{n} K_{h_x}(x - x_i)\, K_{h_y}(y - y_i)\, K_{h_z}(z - z_i)\}, \tag{1}$$

and for regression, we mimic the well-known Nadaraya-Watson estimator,

$$\hat{m}(x, y) = \frac{\sum_{i=1}^{n} z_i K_{h_x}(x - x_i) K_{h_y}(y - y_i)}{\sum_{i=1}^{n} K_{h_x}(x - x_i) K_{h_y}(y - y_i)}, \tag{2}$$

using the ASH.

Now let us change our notation to reflect the use of the ASH to operate on binned data; then

$$x_1, x_2, \ldots, x_{n_x} \qquad y_1, y_2, \ldots, y_{n_y} \qquad z_1, z_2, \ldots, z_{n_z}$$

are the midpoints along each axis of a trivariate mesh of size $n_x \times n_y \times n_z$ with spacings $\delta_x, \delta_y, \delta_z$. Thus

$$\Delta x_i = \delta_x = \frac{h_x}{m_x} \quad \Delta y_i = \delta_y = \frac{h_y}{m_y} \quad \Delta z_i = \delta_z = \frac{h_z}{m_z}$$

for some integers m_x, m_y, m_z and smoothing parameters h_x, h_y, h_z.

Let $\nu_{jk\ell}$ denote the number of data points $(x, y, z)_i$ falling in bin $B_{jk\ell}$. Note that $\sum \nu_{jk\ell} = n$, and we

expect many of the $\nu_{jk\ell}$ to be 0.

The "naive ASH" is constructed by "computing" $m_x \times m_y \times m_z$ (different) trivariate histograms, each with rectangular bin size $h_x \times h_y \times h_z$, but with origins shifted by multiples of $\delta_x, \delta_y, \delta_z$ along the coordinate axes. To be specific, one bin is anchored at the point $(j\delta_x, k\delta_y, \ell\delta_z)$, as j, k, ℓ each range from 0 to n_x-1, n_y-1, n_z-1. The ASH density estimator in bin (j, k, ℓ) is a weighted average of the neighboring bin counts, where the weights are given by

$$w_{abc} = \frac{K\left(\frac{a}{m_x}\right) K\left(\frac{b}{m_y}\right) K\left(\frac{c}{m_z}\right)}{\sum_a \sum_b \sum_c K\left(\frac{a}{m_x}\right) K\left(\frac{b}{m_y}\right) K\left(\frac{c}{m_z}\right)}\,; \tag{3}$$

here, K is supported on $(-1, 1)$ as before, and the indices a, b, c range over $-m_x < a < m_x$, $-m_y < b < m_y$, and $-m_z < c < m_z$. Note that $w_{abc} = w_a w_b w_c$. Furthermore, the weights $\{w_a, w_b, w_c\}$ need only be computed once.

Following the Nadaraya-Watson motivation, the ASH regression estimator is:

$$\hat{m}_{jk} = \frac{\sum_a \sum_b w_{ab} \nu_{j+a,k+b} \bar{z}_{j+a,k+b}}{\sum_a \sum_b w_{ab} \nu_{j+a,k+b}}\,, \tag{4}$$

where $w_{ab} = w_a w_b$.

For the survey sampled data, each data point takes the extended form

$$\{(x, y, z, \alpha)_i\,, \quad i = 1, \ldots, n\}\,,$$

where α_i is the effective sampling weight. Previously, we have assumed that $\alpha_i = 1$ for all cases. In this setting, the frequency counts $\nu_{jk\ell}$ are replaced by the sum of these α_i weights rather than 1's. Furthermore, it is often of interest to include other covariates in the regression analysis, of the form

$$\{(x, y, z, t, \alpha)_i\,, \quad i = 1, \ldots, n\}\,,$$

where t is some covariate of interest. Then we compute the ASH regression estimator $\hat{m}(x, y, t)$ by simply

adding another loop to the numerator and denominator of the $\hat{m}_{jk}$ equation above. The sampling weights are the same of course. Typically, we will map the estimate at several levels of t, for example, $\hat{m}(x, y, t = t_0)$. A fuller derivation of this material may be found in Scott and Whittaker (1996).

3 Applications and Data

Three current applications using the ASH are summarized in this section. A multivariate analysis of agricultural land values demonstrates ASH regression estimation in section 3.1. Estimation and visualization of changes in the distribution of farm profits given an environmental input tax utilizes the ASH density estimator are presented in section 3.2. Finally, a third application illustrates the use of the ASH to link data sets and to link economic models with physical models of the environment in section 3.3.

The use of georeferenced data and geographic information systems (GIS) is discussed in the multivariate and linking applications. The policy applications (density estimation and linking) use the ASH in conjunction with an economic model of agricultural production. The geographical areas the three applications are concerned with are shown in Figure 1.

Data on the physical environment comprise the largest data sets we used, with one set consisting of over 13 million observations on three variables. From the U.S. Geological Survey (USGS), we used digital elevation models (DEM), records of terrain elevations for ground positions at regularly spaced horizontal intervals; hydrologic unit maps of the United States, which show the boundaries of river basins; and lcc159, a one kilometer grid of land use-land cover for the conterminous United States based on satellite imagery (Loveland, et al., 1991). These data are available on the USGS website.

The STATSGO data set is a digital general soil association map developed by the National Cooperative Soil Survey and distributed by the Natural Resources Conservation Service (formerly Soil Conservation Service) of the U.S. Department of Agriculture. It consists of a broad-based inventory of soils and non-soil areas that occur in a repeatable pattern on the landscape. STATSGO is available on the NRCS website.

Data on population are from the 1990 census at the block group level. These data are available in ARC/INFO export format on the USGS website.

Economic data on farm operations were obtained from the Farm Costs and Returns Survey (FCRS), a sur-

vey jointly administered by the National Agricultural Statistics Service (NASS) and the Economic Research Service (ERS) of the U.S. Department of Agriculture. The FCRS sample is drawn without replacement from stratified area and list frames of farm operations. Several thousand usable questionnaires with over three hundred variables are received each year.

The land values were taken from the June Agricultural Survey, a stratified survey representing farmer's estimates of the current market value of specific tracts they operate (Barnard and Westenbarger, 1995). This survey is conducted by NASS. All of the survey data are subject to federal law on confidentiality, and are restricted in availability.

3.1 Multivariate Analysis of Land Values

Land values are important to agriculture because land provides the major asset value which supports agricultural production. Visualization and analysis of land values is consequently important in agricultural policy analysis. By linking nonparametric regression to GIS mapping software, we are able to provide a unique perspective on the spatial distribution of farmland values.

In a study of land values in the upper Midwest, we were interested in the effect of population and direct government payments on agricultural land values. Figure 2 displays an ASH estimate of crop land values per acre as a function of longitude and latitude. Even a cursory look at the spatial distribution of land values confirms the common knowledge that land values increase as the urban core is approached (note particularly the area by Chicago). The first part of the analysis was to re-estimate the value of land conditioned on low population density, *i.e.*, with the effect of urban levels of population density removed. Figure 3 shows an ASH surface of the population density in the study area. The bin size is quite small, and is clearly visible in the figure (since we performed relatively little smoothing). This figure is offered to show the correlation between land value and population density, although a closer examination suggests the relationship differs in different areas. The estimate of the value of crop land given a rural level of population (1,500 persons or less per square mile) is presented in Figure 4, which retains the largest features of the estimate of crop land values. The highest value drops from $10,000 to $4,000 per acre; the large, local peaks due to urban influence have been effectively diminished. The Chicago area remains the highest value, but it turns out

that Chicago is also located on the most productive land in the Midwest.

To complete the analysis, we re-estimated land values as a function of longitude, latitude, population density, and direct government payments per acre. Direct government payments are made to farmers under government programs designed to support and stabilize commodity prices. An ASH estimate of direct government payments per acre alone is shown in Figure 5. To calculate the fraction of agricultural land value due to direct government payments, we took a 4-dimensional slice of the five-dimensional estimation space where population density was limited to less than 1,500 persons per square mile. Next, we extracted two 3-dimensional slices from this subset, where the slices are limited in the government payment dimension to ranges of 9 to 15 dollars per acre and 15 to 21 dollars per acre. These slices represent low and high levels of direct government payments per acre, and their difference is shown in Figure 6. Clearly, land values are positively correlated with the value of government payments. The difference in these two slices divided by land value given a low level of direct government payments results in an estimate of the fraction of land value due to high direct government payments per acre (Figure 7).

The ASH estimation readily lends itself to visualization with GIS software. The results of the estimation provide a value for the center of each bin. These values are easily imported in software such as ARC/INFO or ArcView, and they tile a map. Boundaries and other features available in geographic data bases can then be added to help the interpretation. Our estimates usually extend beyond the boundaries of the study area, as we have chosen not to employ boundary correction techniques (see Scott and Whittaker, 1996). The GIS software is used to clip the edges of the estimate to match the boundaries, on the assumption that the level of the estimate shows no unusual behavior near the edges.

3.2 Change in Distribution of Profits

Policy analysis in production economics is commonly approached by building a model of the production technology, then comparing the effects of different policies on the model. In the applications discussed here, we chose to apply data envelopment analysis (DEA) to model the production technology of farms (see Whittaker, 1994, and Färe and Whittaker, 1995, for examples of application of DEA to agricultural production). DEA is a nonparametric approach in the sense that no assumptions are made about the

functional relationships among variables. The solution to a DEA model provides a measure of the "best-practice" technology, *i.e.*, the highest profit that each firm could possibly achieve.

In the DEA model used here, there are $k = 1, \ldots, K$ observations of farms. Each farm uses $x = (x_1, \ldots, x_M) \in \Re_+^M$ inputs to produce $u = (u_1, \ldots, u_N) \in \Re_+^N$ outputs. The observed inputs $x^k = (x_1^k, \ldots, x_M^k)$ and the observed outputs $u^k = (u_1^k, \ldots, u_N^k)$ are used together with the intensity variables $z^k \geq 0$, $k = 1, \ldots, K$, to form the reference technologies. Our basic model is then

$$\begin{aligned} T = \{(x,u) : u_n &\leq \sum_{k=1}^{K} z^k u_n^k, && n = 1, \ldots, N, && (5.1)\\ x_m &\geq \sum_{k=1}^{K} z^k x_m^k, && n = 1, \ldots, M, && (5.2)\\ & \sum_{k=1}^{K} z_k = 1, && z_n \geq 0\}. && (5.3) \end{aligned}$$

Denote input prices by $p^k \in \Re_+^M$ and output prices by $r^k \in \Re_+^N$. Then the profit of farm k can be computed as the solution to the following linear programming problem:

$$\begin{aligned} \pi(r^k, p^k) \;=\; max & \sum_{n=1}^{N} r_n^k u_n - \sum_{m=1}^{M} p_m^k x_m && && && (6.1)\\ s.t. \quad & \sum_{k=1}^{K} z^k u_n^k \;\geq\; u_n, && && n = 1, \ldots, N && (6.2)\\ & \sum_{k=1}^{K} z^k x_m^k \;\leq\; x_m, && && m = 1, \ldots, M && (6.3)\\ & \sum_{k=1}^{K} z^k \;=\; 1 && && && (6.4)\\ & z_k \;\geq\; 0, && && k = 1, \ldots, K \;\; . && (6.5) \end{aligned}$$

To apply an input tax on the Mth input, for example, the objective function (6.1) becomes

$$\pi(r, p, tax) = max \sum_{n=1}^{N} r_n u_n - \sum_{m=1}^{M-1} p_m x_m - x_M(p_M + tax), \qquad (7)$$

where *tax* is the tax rate. Taxes on multiple inputs are represented by additional terms in the objective function.

The motivation of this application of the ASH is to investigate the economic effects of mitigation of nitrate pollution in water on the Columbia Plateau. Elevated levels of nitrate have been observed in ground and surface water in that area, and are attributed to agricultural fertilizer (Bortleson, 1991).

Taxes of various sorts have been proposed to mitigate the effects of non-point source pollution from agriculture, including taxes on emissions, inputs, and outputs. Taxes on inputs are by far the easiest to enforce and monitor (Schmutzler and Goulder, 1997). Therefore we chose to analyze an input tax on nitrogen. The effects on farm profits of an imposition of multiple levels of an input tax was modeled in the DEA profit maximizing framework described above. The ASH version of density estimation (Scott, 1992) was used to visualize the distribution of profits under different tax schemes. The results displayed in Figure 8 show that all farms are affected, but profits of small (as measured by gross cash farm income) and large farms are reduced proportionately more than medium size farms.

3.3 Linking Data Sets and Models

It is a common situation to find that data sets which ostensibly refer to the same geographic region, are in fact measured at different levels of aggregation and different geographical sub-regions. For example, in the crop land values study discussed above, population density is known at the census block group level, direct government payments at the county level, and crop land values at scattered points. To link these data sets for analysis, we need find a set of points where a value can be calculated for each variable. This process is relatively easy using the ASH regression estimator. One simply bins each data set to the same geographic grid, then operates on the binned data as usual.

In our final application, we take advantage of this ability to link data sets to link economic and physical models to effect a comprehensive environmental policy analysis. Non-point pollution sources from agriculture have been identified as contributing large amounts of nitrogen to both ground and surface water in the Columbia River Basin of Oregon, Washington, and Idaho (Greene et al., 1994). A tax on nitrogen fertilizer application is one of the so-called incentive measures for the reduction of non-point source pollution. In this application, we linked the DEA models of nitrogen input taxation to an environmental model to estimate physical and economic effects of the tax regime.

The Soil and Water Analysis Tool (SWAT) was used to model the nitrogen loadings in water based on results from the economic models. SWAT is a simulation model for large basins of tens of thousands of square kilometers. SWAT is physically based, since calibration is not possible in large, ungaged basins. The components of SWAT include weather, surface runoff, return flow, percolation, crop growth, irrigation, groundwater flow, reach routing, and nutrient and pesticide loading, among other features (Srinivasan and Arnold, 1994; Srinivasan, 1995). SWAT also provides a very complete set of variables to simulate different management practices. SWAT runs in a GIS system and uses raster maps as input data sets. The ASH is used to link the economic models and SWAT by giving SWAT a surface of inputs from the economic model (see for example, Figure 9) for each physical variable. Thus, human economic behavior drives the physical model through the ASH surfaces. SWAT requires three data sets describing the physical geography of a study area in order to model an area. The data sets are entered into the model in the form of raster maps, which are used by a GIS interface to SWAT. The data used in this application (consisting of several million observations) are land use (Figure 10), soil (Figure 11), and topography (Figure 12).

To analyze the nitrate tax policy, we solved the models (6) and (7) for comparison of different levels of taxation. The results from (6) and (7) provided points to estimate surfaces of nutrient applications and yields. The surfaces were used to estimate the mode of each variable in each watershed. The final step was to run SWAT and compare the costs to producers and environmental benefits of a nutrient tax. A tax rate of 300% was about the lowest level of taxation which produced a reduction in nutrient application.

One result from this analysis is that there is a large variation in effective tax burden within the study area. The amount per acre profits are reduced by the imposition of a 300% tax vary from \$0 to \$142.03 throughout the region (Figure 13). As a measure of the efficiency of the 300% tax in nitrogen reduction, we calculated the cost of a 1 kilogram per hectare reduction in nitrogen reaching the mouth of the watershed (Figure 14). Again, there is large variability, although there seems to some correlation with nitrogen application rate (compare Figure 9 and Figure 14).

4 Conclusion

The averaged shifted histogram has proven to be a versatile, computationally efficient technique in several different applications in environmental economics. The ASH density and nonparametric regression estimators were able to accommodate economic data sets from complex, stratified surveys, with unknown distributions. The ASH made the estimation and visualization of model results possible for previously intractable problems in linking economic behavior to environmental processes. Output from the ASH estimators is suitable for geographic information system software, which provides greatly enhanced visualization for geographically referenced data.

Acknowledgments

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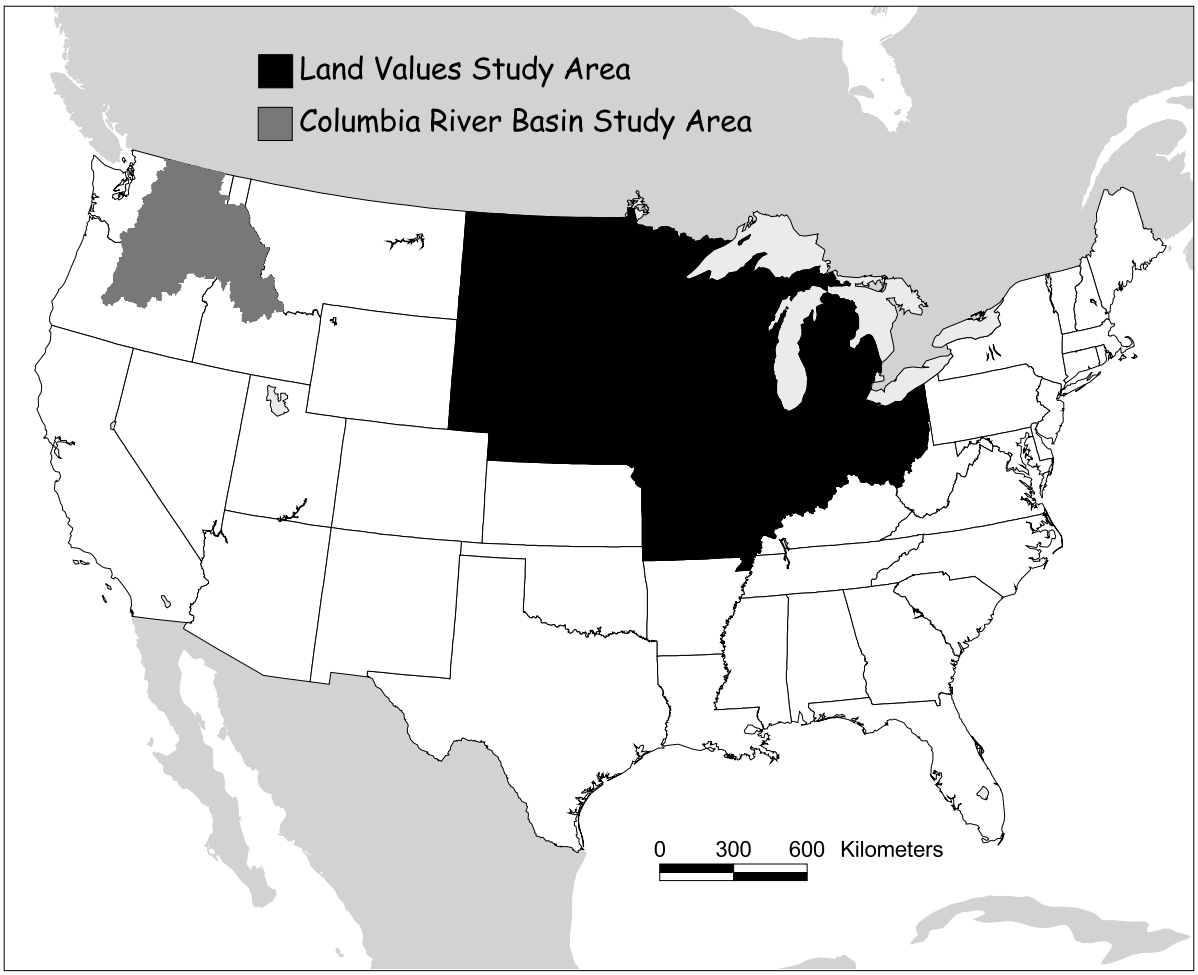

Figure 1: Areas of study for the applications discussed in this paper.

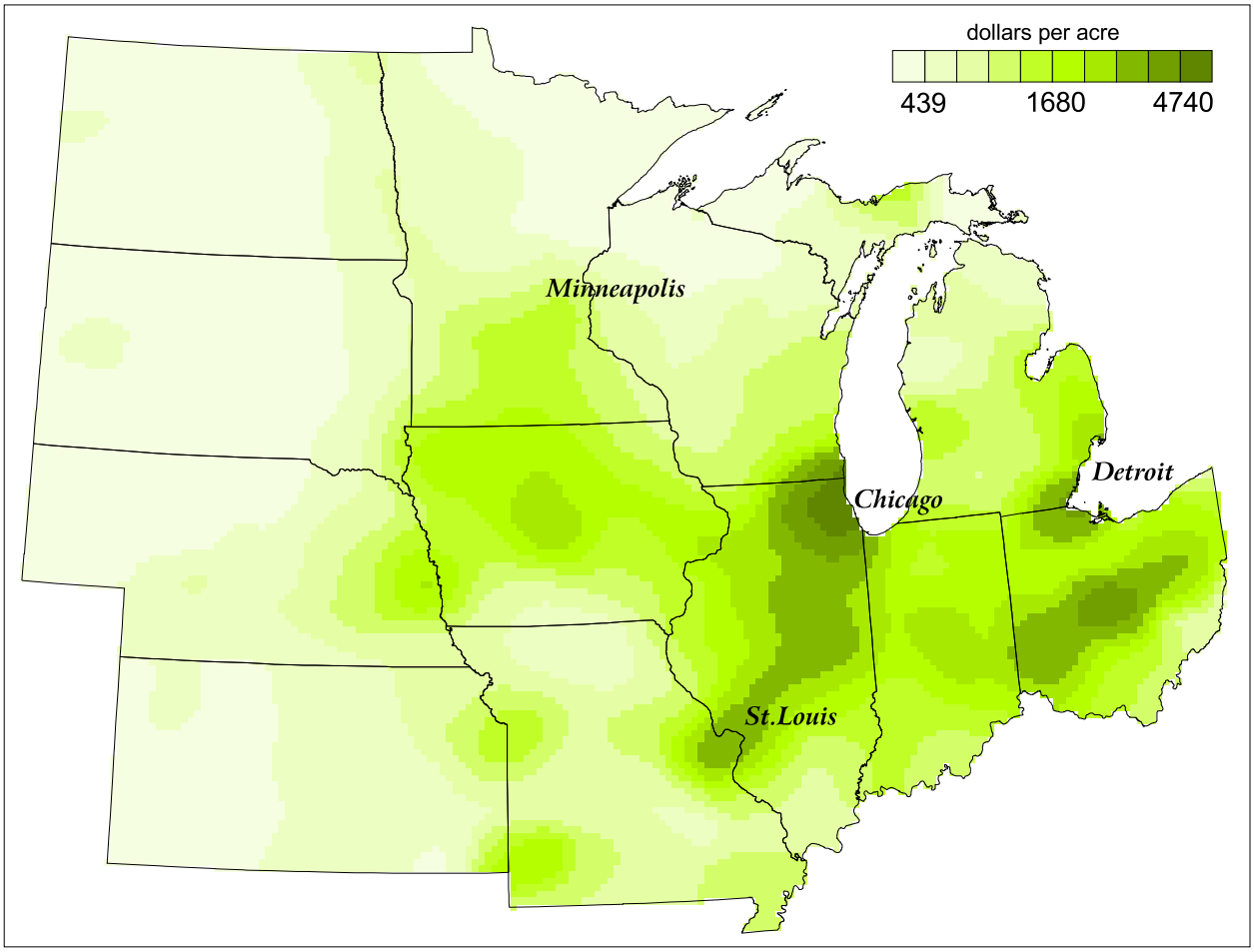

Figure 2: Crop land values.

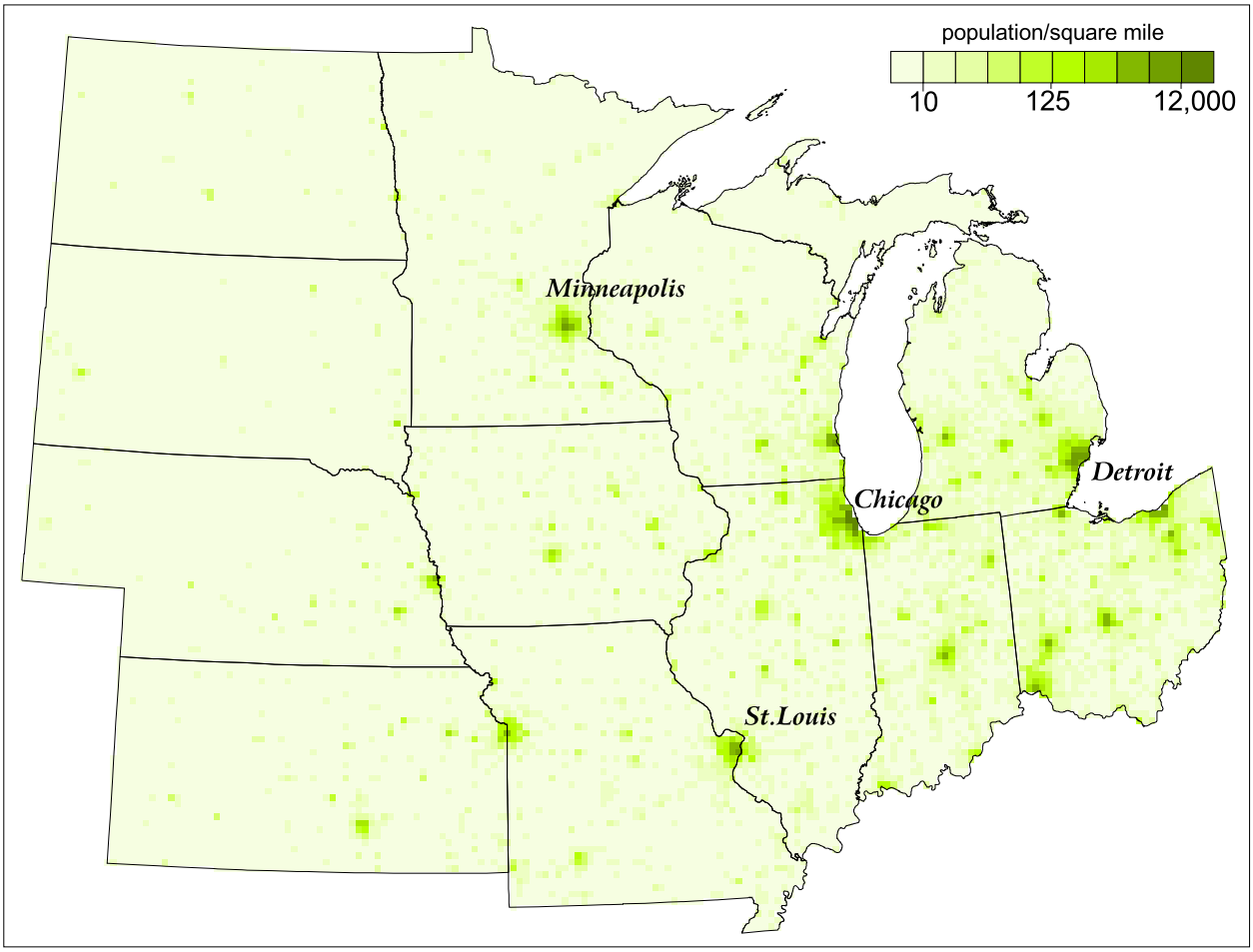

Figure 3: Population density.

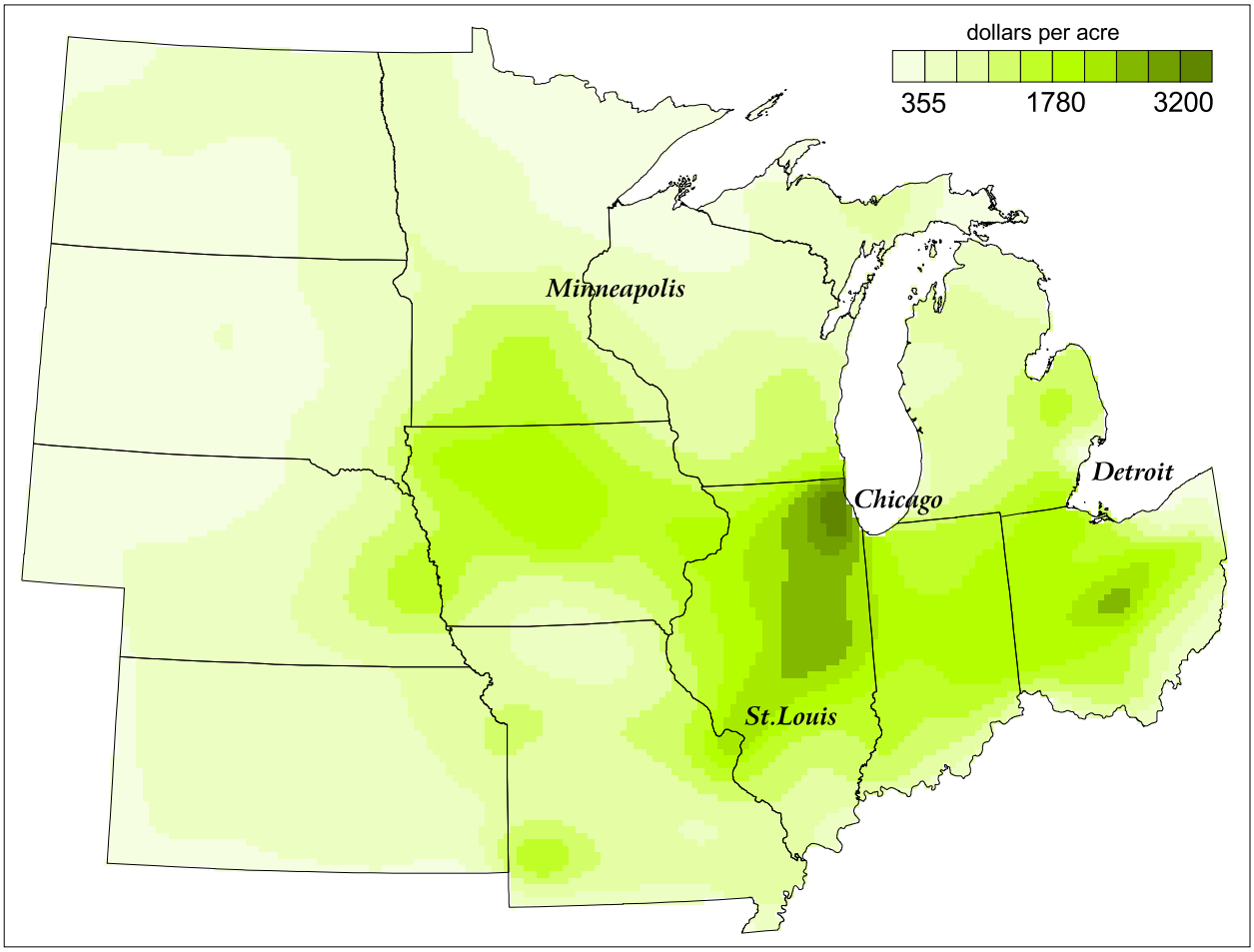

Figure 4: Land values conditioned on a rural level of population.

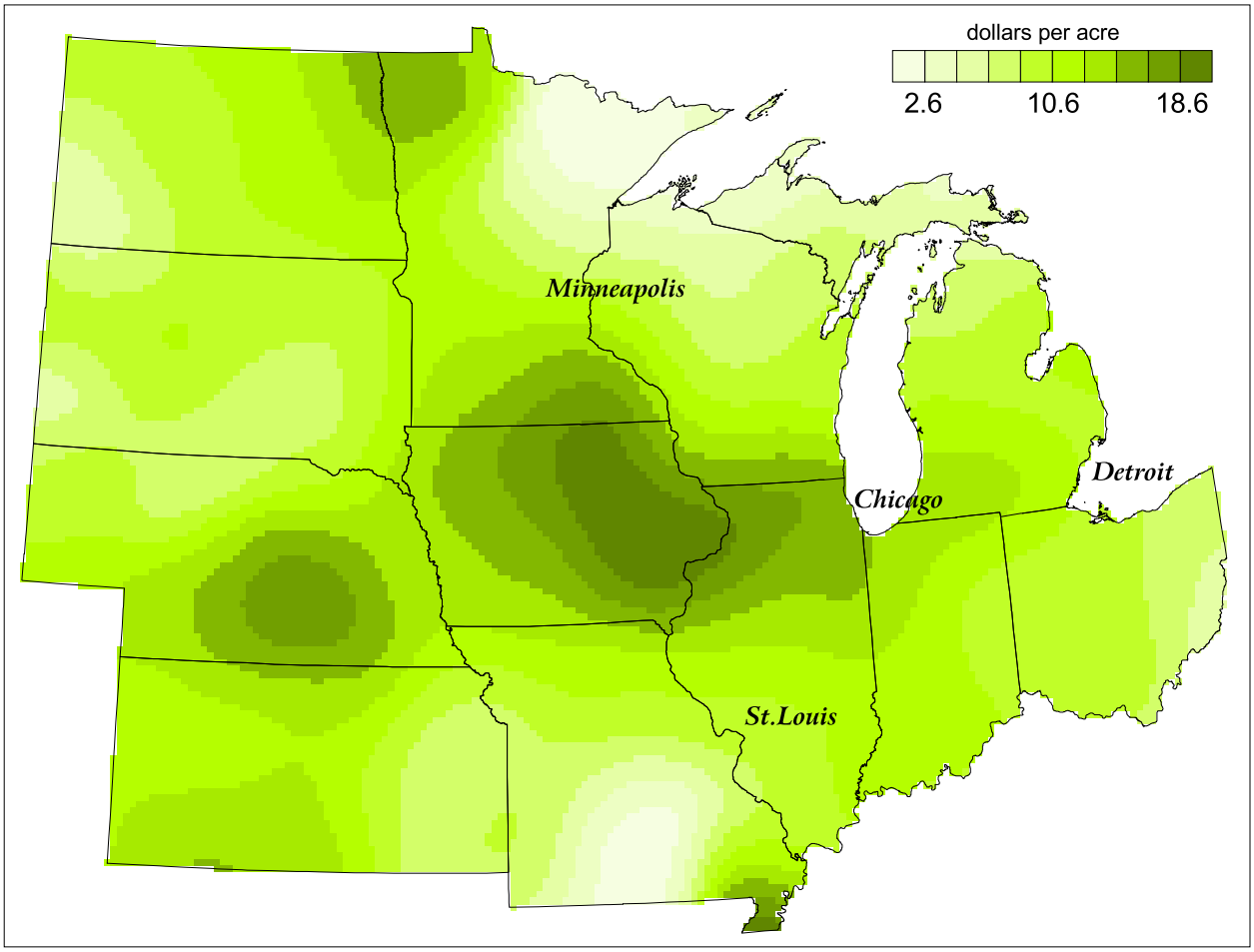

Figure 5: Direct government payments per acre.

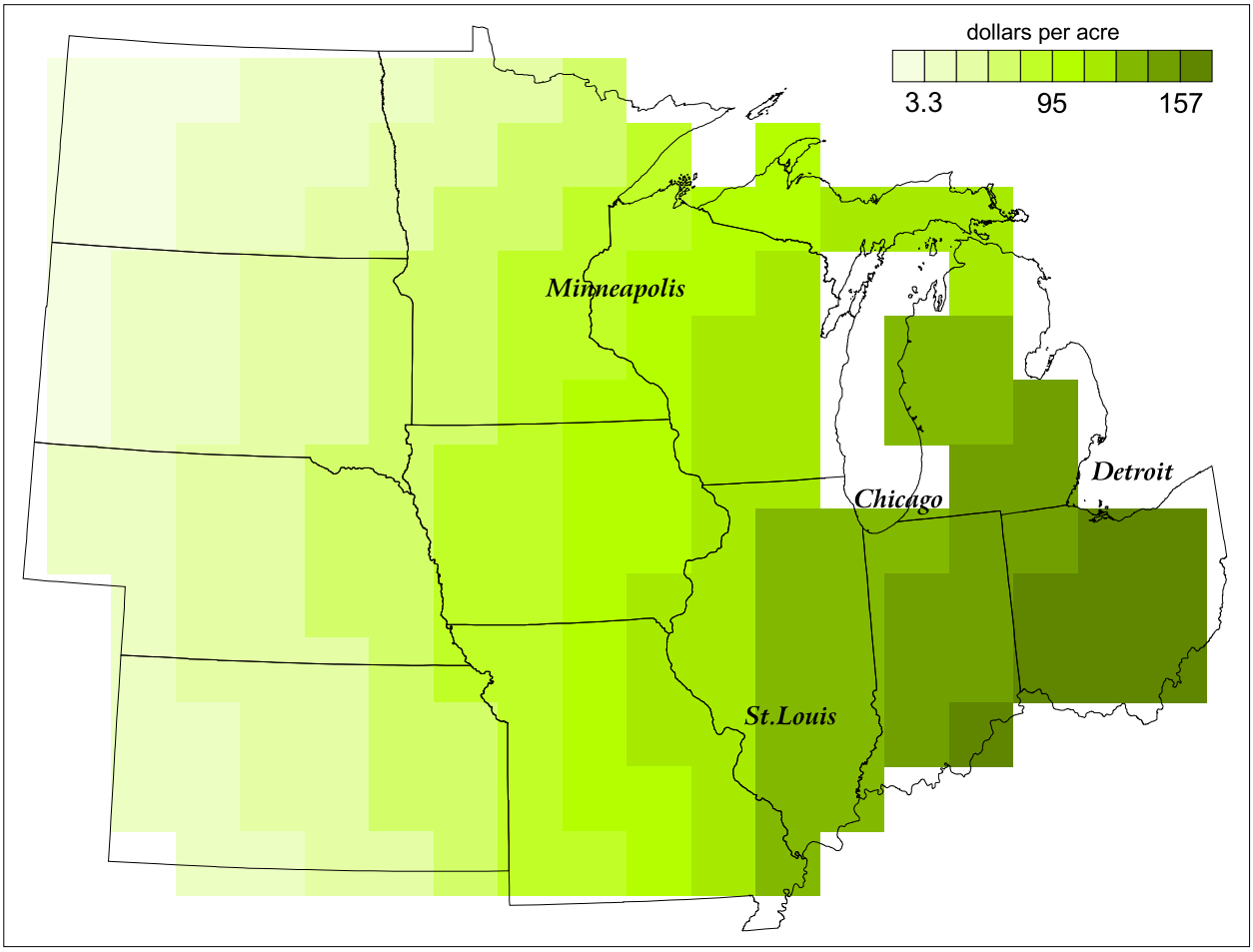

Figure 6: Difference of land values conditioned on latitude longitude, rural population, and two levels of government payments (¡ $5/acre, =>5/$acre.

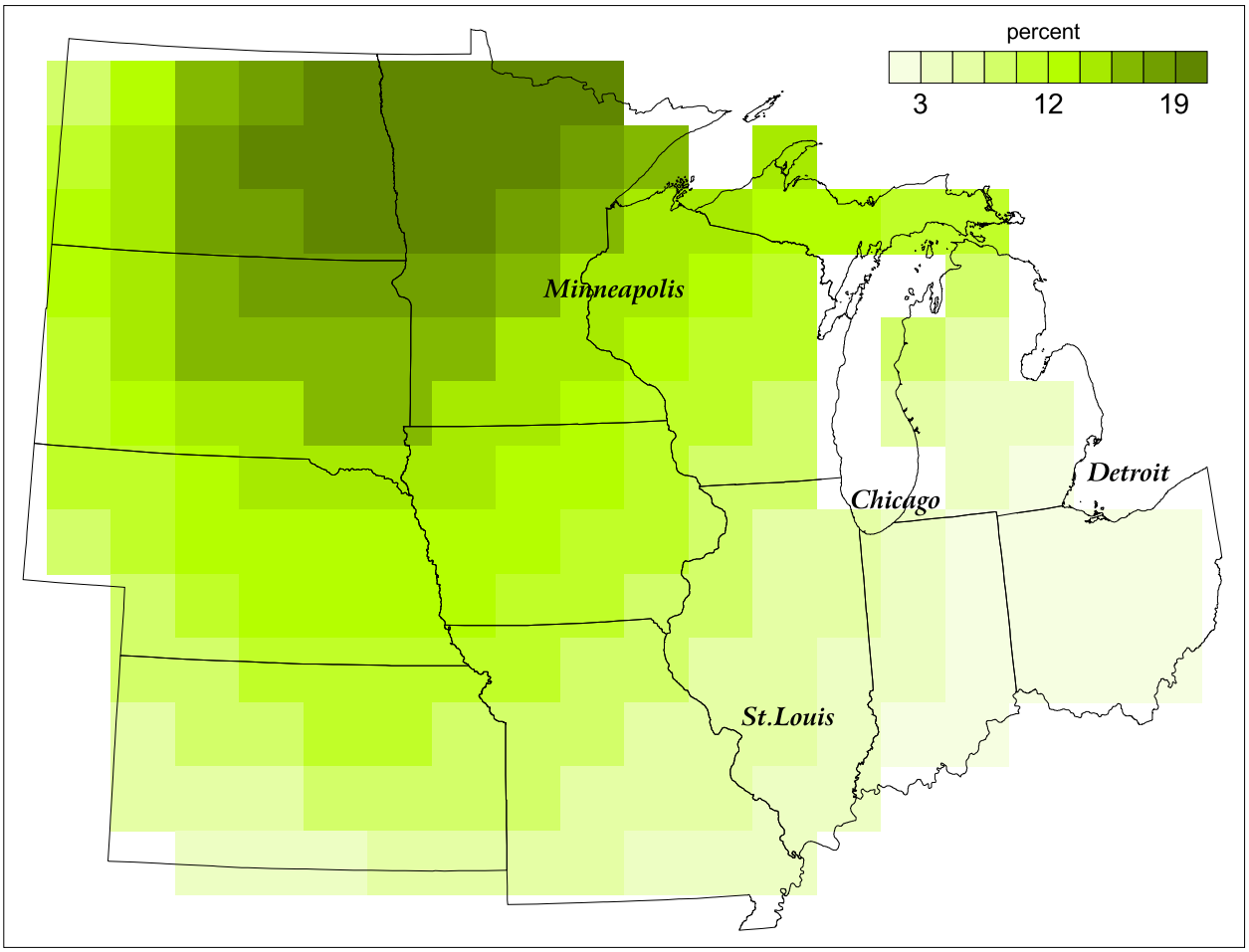

Figure 7: Proportion of crop land value due to direct government payments.

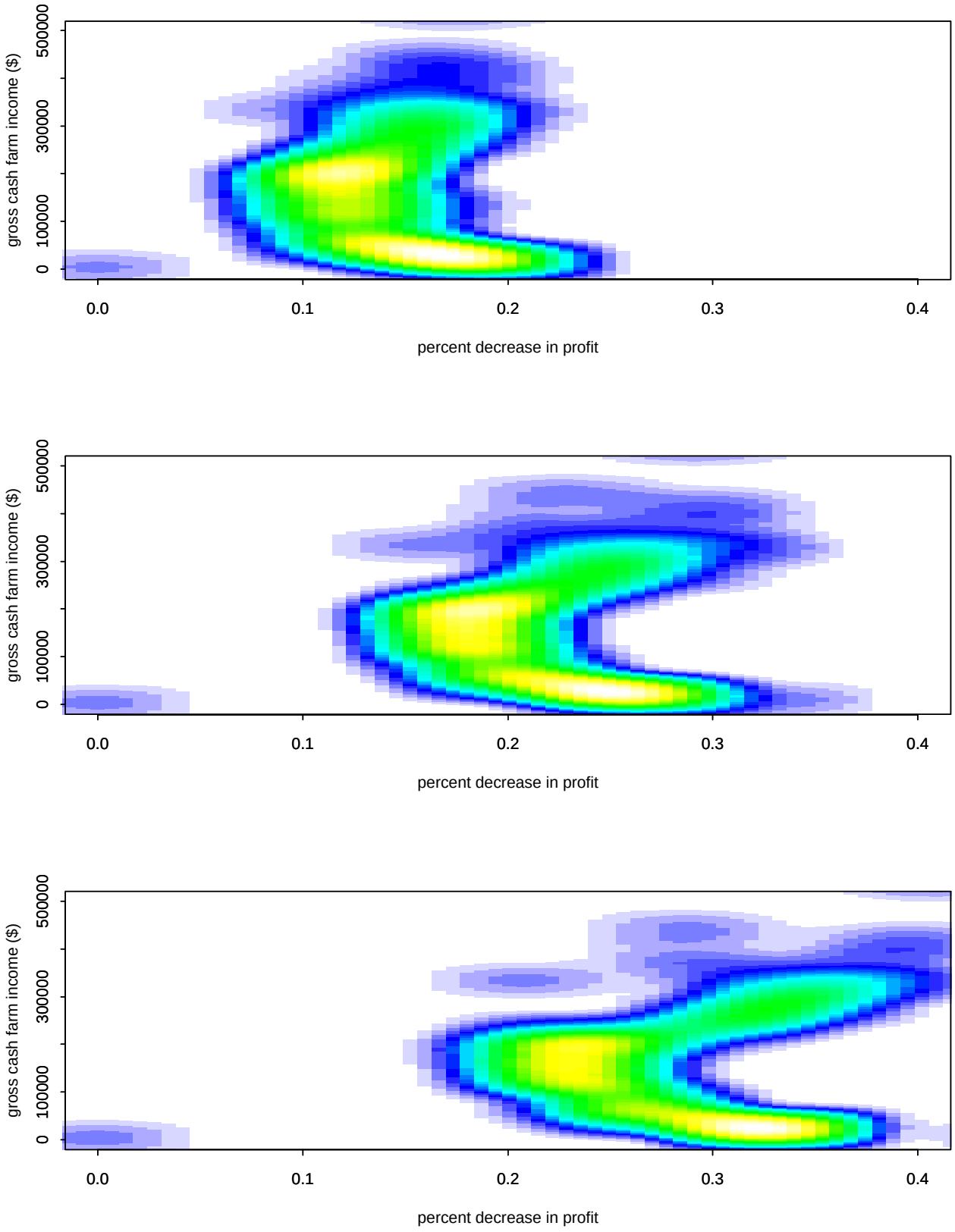

Figure 8: Predicted changes in distribution of profits at different levels of nitrogen taxation. Top: Imposition of 300% tax, middle: 500% tax, bottom: 700% tax.

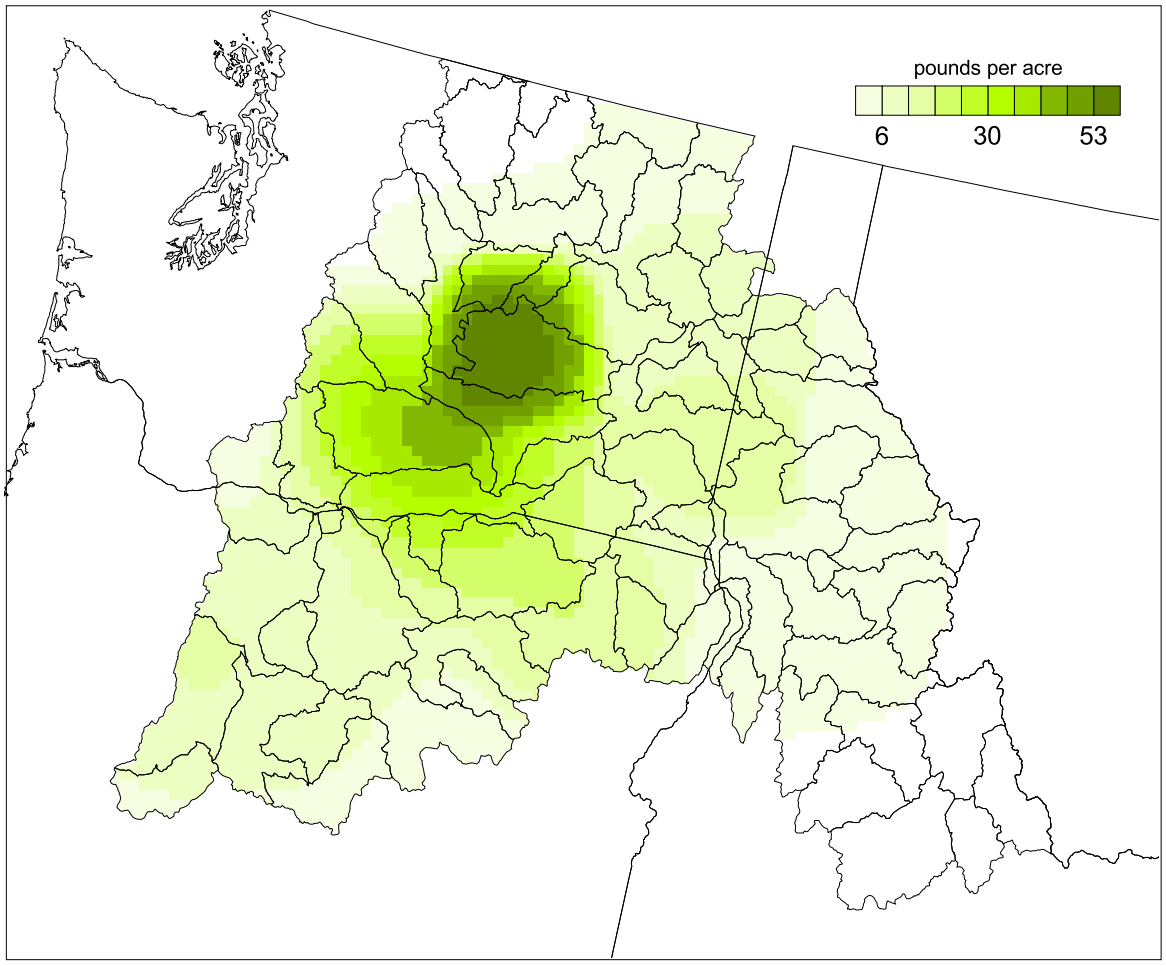

Figure 9: Surface of nitrate application to wheat, optimum predicted by economic model. State and 8-digit hydrologic units boundaries are shown.

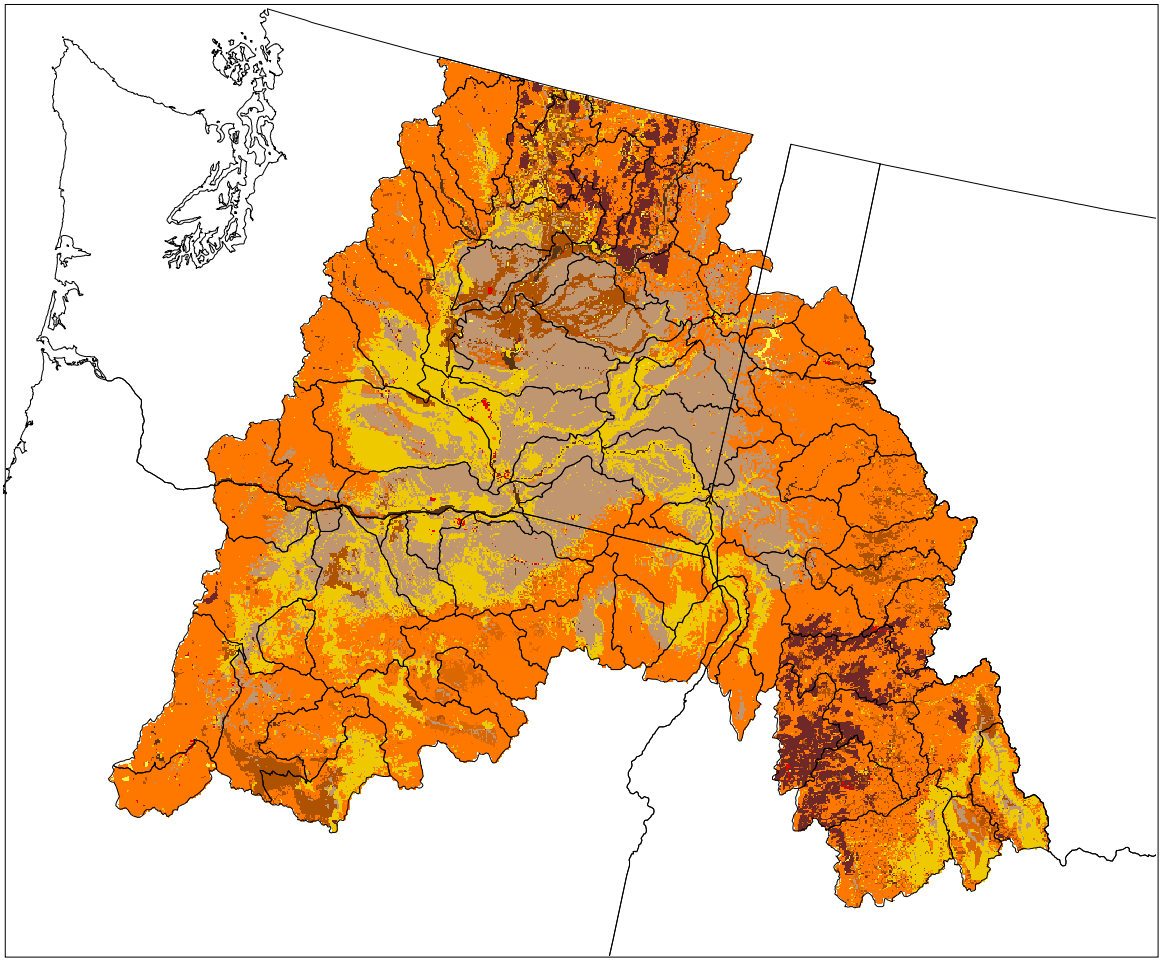

Figure 10: Land use map of the the Columbia Plateau, input to SWAT.

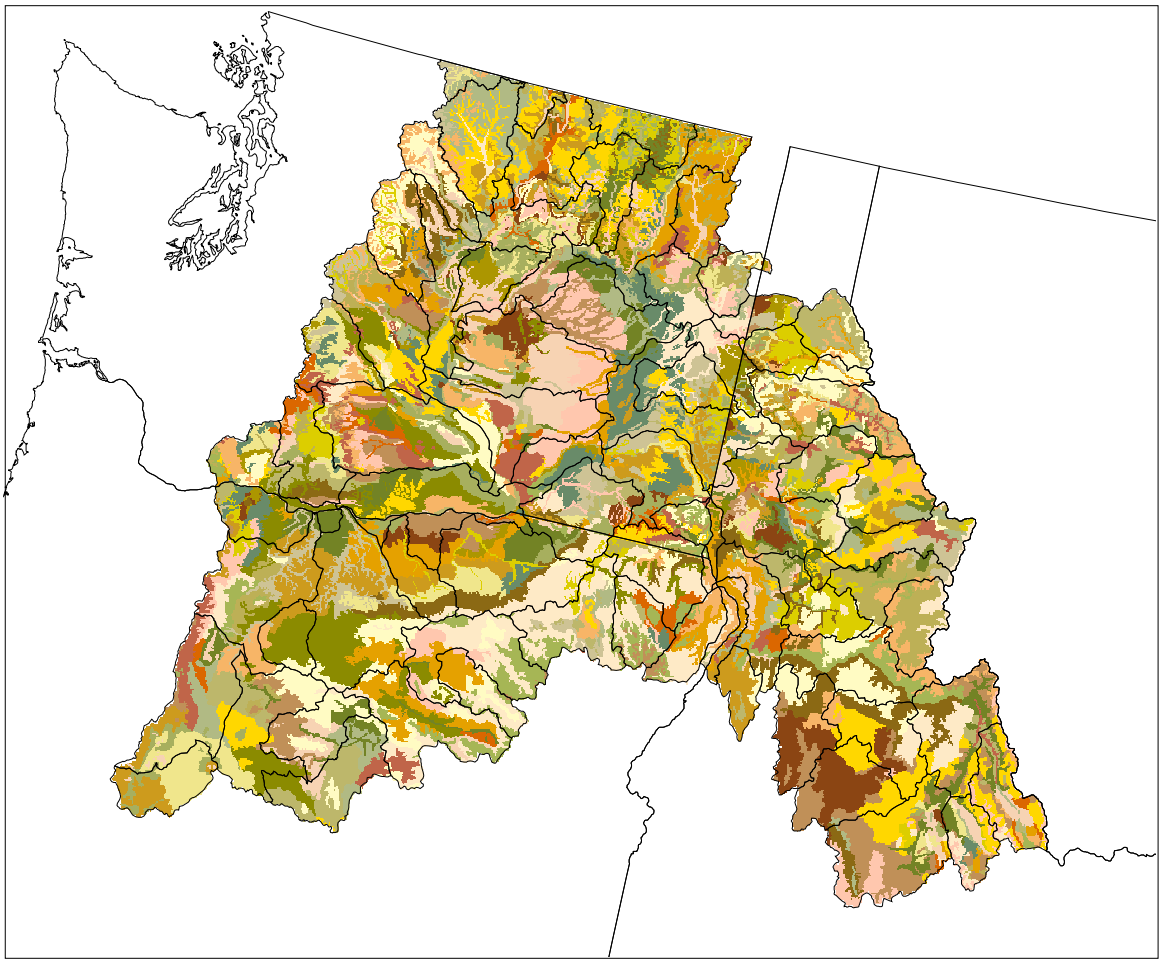

Figure 11: Soils map of the Columbia Plateau, input to SWAT.

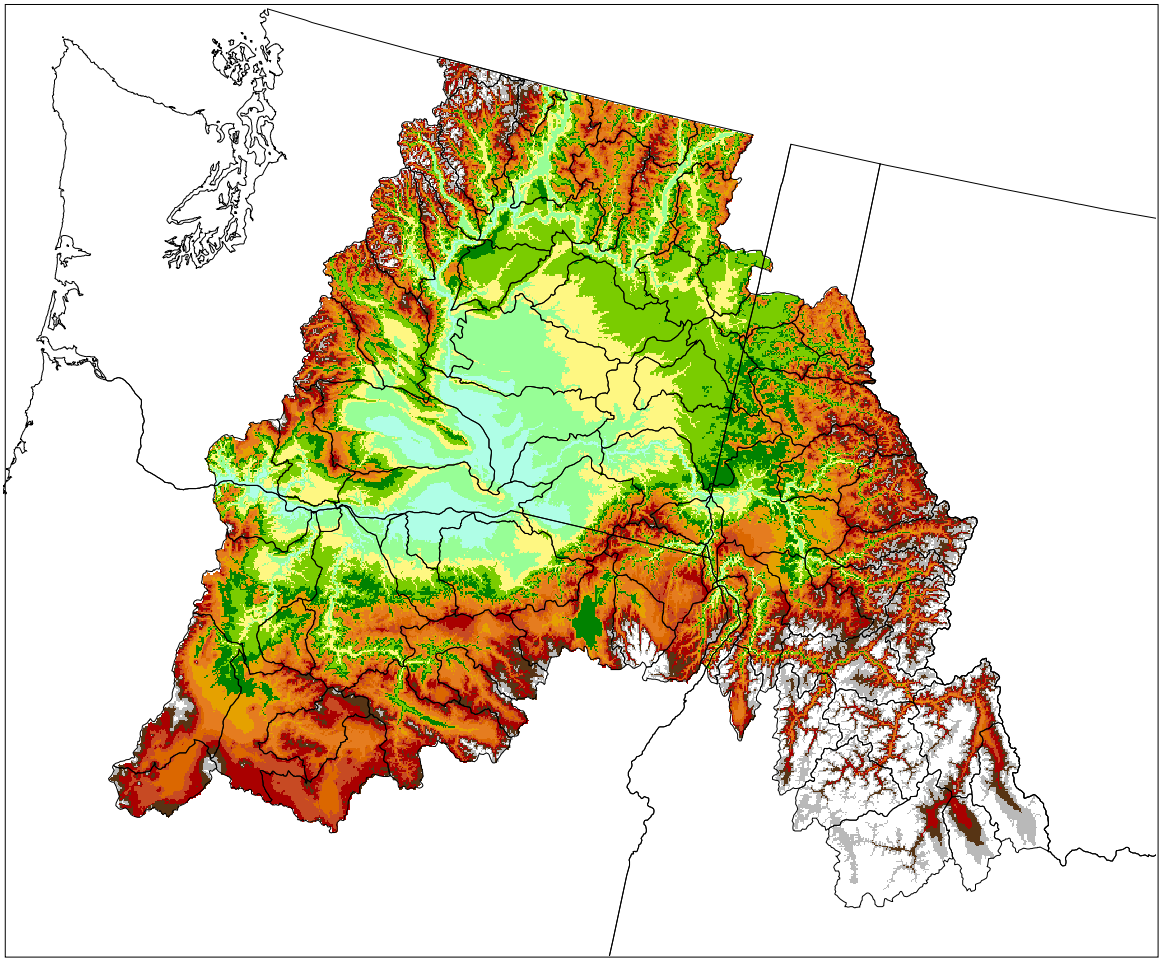

Figure 12: Digital elevation model of the Columbia Plateau, input to SWAT.

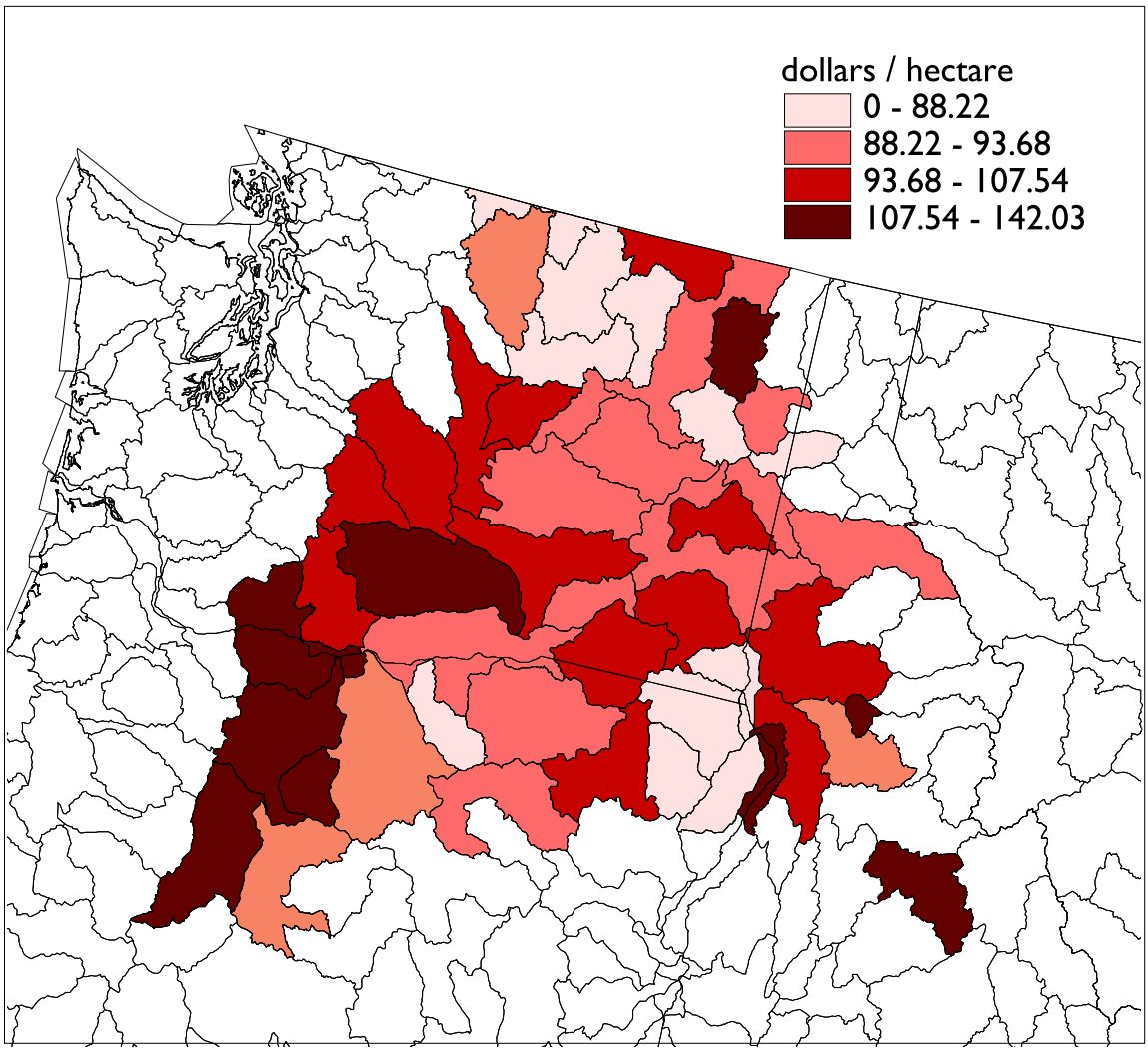

Figure 13: Reduction in profit resulting from a 300% input tax on nitrogen fertilizer.

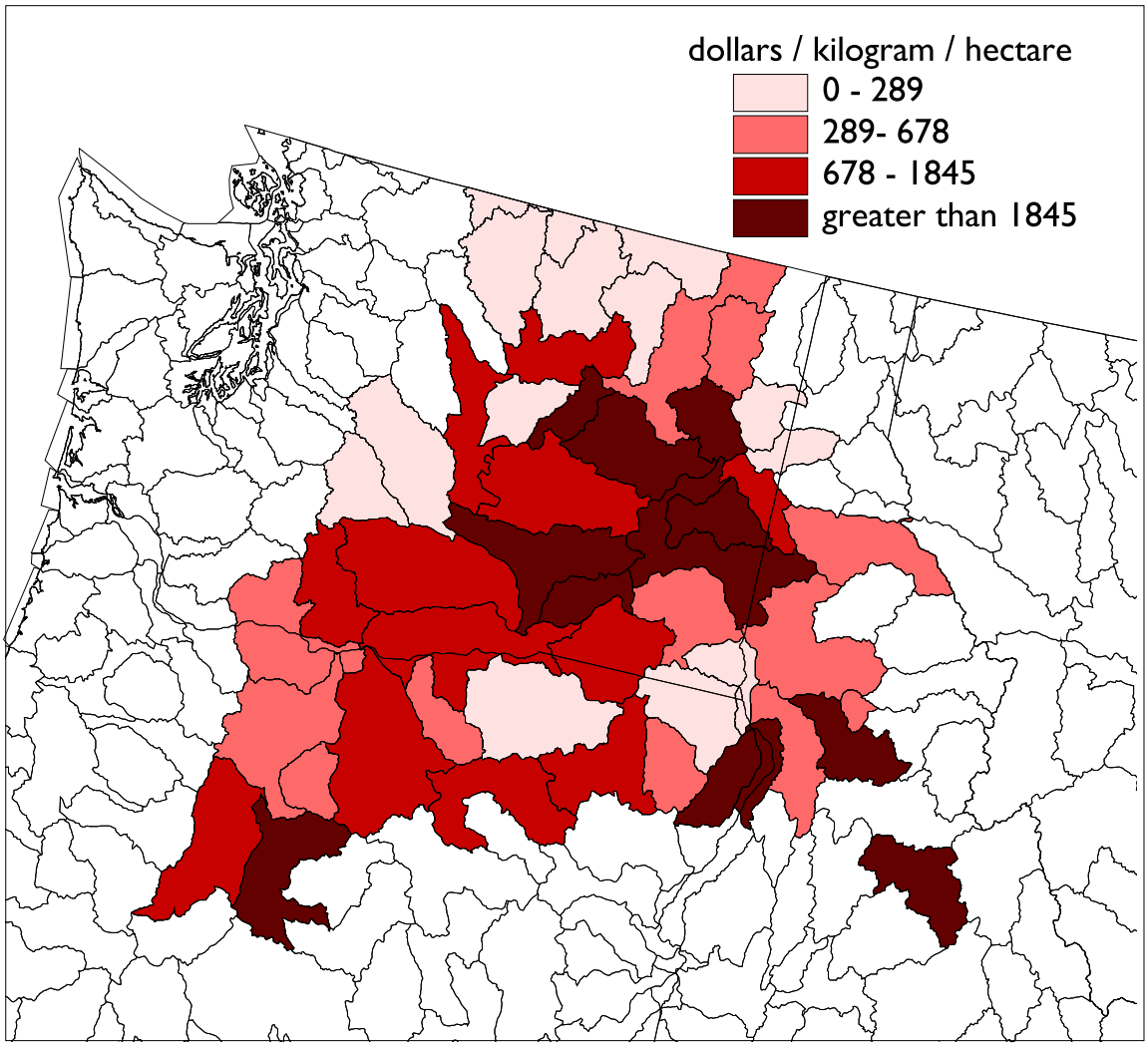

Figure 14: Mean surface water tax efficiency of 300% input tax on nitrogen fertilizer.