Stat 550 Virtual Whiteboard

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September 5, 2023

$$f(y) \approx f(x) + (y-x)\underline{\underline{f'(x)}} + \frac{1}{2}(\;)^2 f''(x)$$

$$ISE(\hat\theta) = \int \left[f_{\hat\theta}(x) - f_\theta(x)\right]^2 dx \qquad \frac{d}{dy} f(y)\Big|_{y=x}$$

$$I(\hat\theta) = I(\theta) + (\hat\theta - \theta)\frac{d}{d\hat\theta} I(\hat\theta)\Big|_{\hat\theta=\theta}$$

$$+ \frac{1}{2}(\hat\theta - \theta)^2 \frac{d^2}{d\hat\theta^2} I(\hat\theta)\Big|_{\hat\theta=\theta} + \dots$$

$$I(\theta) = 0 \qquad \frac{dI(\hat\theta)}{d\hat\theta}\Big|_{\hat\theta=\theta} = 0$$

$$E\left[I(\hat\theta)\right] = \mathrm{Var}\,\hat\theta \cdot \text{junk}$$

eg $N(\bar{x}, v)$ $(\hat{\theta} = \bar{x})$

$$f_{\hat{\theta}}(x) = \frac{1}{\sqrt{2\pi}\sqrt{v}} e^{-\frac{1}{2}\frac{(x-\bar{x})^2}{v}}$$

$$e^{-\frac{1}{2}\frac{(x-\hat{x})^2}{v}}$$

$$\frac{d}{d\theta} = \frac{x-\bar{x}}{\sqrt{2\pi}\sqrt{v}\; v} e^{-\frac{(x-\bar{x})^2}{v}}$$

$$[\;]^2 = \frac{(x-\bar{x})^2}{2\pi v^3} e$$

$$\int 2 \cdot [\;]^2 \ldots$$

$$\mathrm{MISE}(\bar{x}-\theta) = \frac{\sigma^2}{n} \cdot \frac{1}{4\sqrt{\pi}\sigma^3} = \frac{1}{4\sqrt{\pi}\sigma n} + \ldots$$

Detail

$$I(\theta) = \int f_\theta(x)^2 - 2 f_{\theta_0}(x) f_\theta(x) + f_{\theta_0}(x)^2$$

$$\frac{dI}{d\theta} = \int 2 f_\theta \frac{d f_\theta}{d\theta} - 2 f_{\theta_0} \frac{d f_\theta(x)}{d\theta} + 0$$

$$\frac{d^2 I}{d\theta^2} = \int 2\left[\frac{d f_\theta}{d\theta}\right]^2 + 2 f_\theta \frac{d^2 f_\theta}{d\theta^2} - 2 f_{\theta_0} \frac{d^2 f_\theta}{d\theta^2}$$

$$N(m, S2)$$

$$f_{\hat\theta}(x) = \frac{1}{\sqrt{2\pi}\sqrt{S2}}\, e^{-\frac{1}{2}\frac{(x-m)^2}{S2}}$$

$$[\ \ ]^2 \Big|_{S^2=\sigma^2} = \frac{3}{32\sqrt{\pi}\, S2^{3/2}}\Big|_{S2=\sigma^2}$$

$$mSE(S^2) = Var\, S^2 \cdot \frac{3}{32\sqrt{\pi}\,\sigma^5} \qquad \frac{2\sigma^2}{n-1}$$

$$\frac{3}{16\sqrt{\pi}\,\sigma(n-1)}\ !!$$

$$I(\hat{\theta}_1, \hat{\theta}_2) = I(\theta_1, \theta_2) +$$

$$(\hat{\theta}_1 - \theta_1)\frac{\partial}{\partial \theta_1} \qquad (\hat{\theta}_2 - \theta_2)\frac{\partial}{\partial \theta_2}$$

$$\frac{1}{2}(\hat{\theta}_1 - \theta_1)^2 \frac{\partial^2}{\partial \theta_1^2} \qquad \frac{1}{2}(\hat{\theta}_2 - \theta_2)^2 \frac{\partial^2}{\partial \theta_2^2}$$

$$(\hat{\theta}_1 - \theta_1)(\hat{\theta}_2 - \theta_2)\frac{\partial^2}{\partial \theta_1 \partial \theta_2}$$