Stat 550 Virtual Whiteboard

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Asymptotic MISE for Histogram

$$\boxed{\text{AMISE}(h, n) = \frac{1}{nh} + \frac{1}{12} h^2 R(g')}$$

The first term in . . . is

$$-\frac{R(g)}{n},$$

no h

which does not involve the bandwidth h.

eg. $g \sim N(\mu, \sigma^2)$

$$R(g') = \frac{1}{4\sqrt{\pi}\sigma^3}$$

$$\text{AMISE}(h, n) = \frac{1}{nh} + \frac{1}{48\sqrt{\pi}\sigma^3} h^2 .$$

h_n^* for a Histogram

$$\text{AMISE}(h,n) = \frac{1}{nh} + \frac{1}{12} h^2 R(g')$$

$$\frac{\partial \text{AMISE}(h,n)}{\partial h} = \frac{-1}{nh^2} + \frac{1}{6} h R(g')$$

$$= 0 \quad \text{at } h = h_n^* \qquad \Longrightarrow$$

$$\boxed{(h_n^*)^3 = \frac{6}{nR(g')}}$$

if $g \sim N(\mu, \sigma^2)$

$$(h_n^*)^3 = \frac{24\sqrt{\pi}\sigma^3}{n}$$

$$\boxed{h_n^* \approx 3.49083\, \sigma\, n^{-1/3}}$$

$$\boxed{\hat{h}_n^* = 3.5\, s_x\, n^{-1/3} \qquad \text{Scott's Rule}}$$

$$\text{AMISE}(h_n^*, n) = \frac{3^{2/3}}{4\,\pi^{1/6}\,\sigma} n^{-2/3} = \frac{0.620}{\sigma}\, n^{-2/3}.$$

$h \to 0$

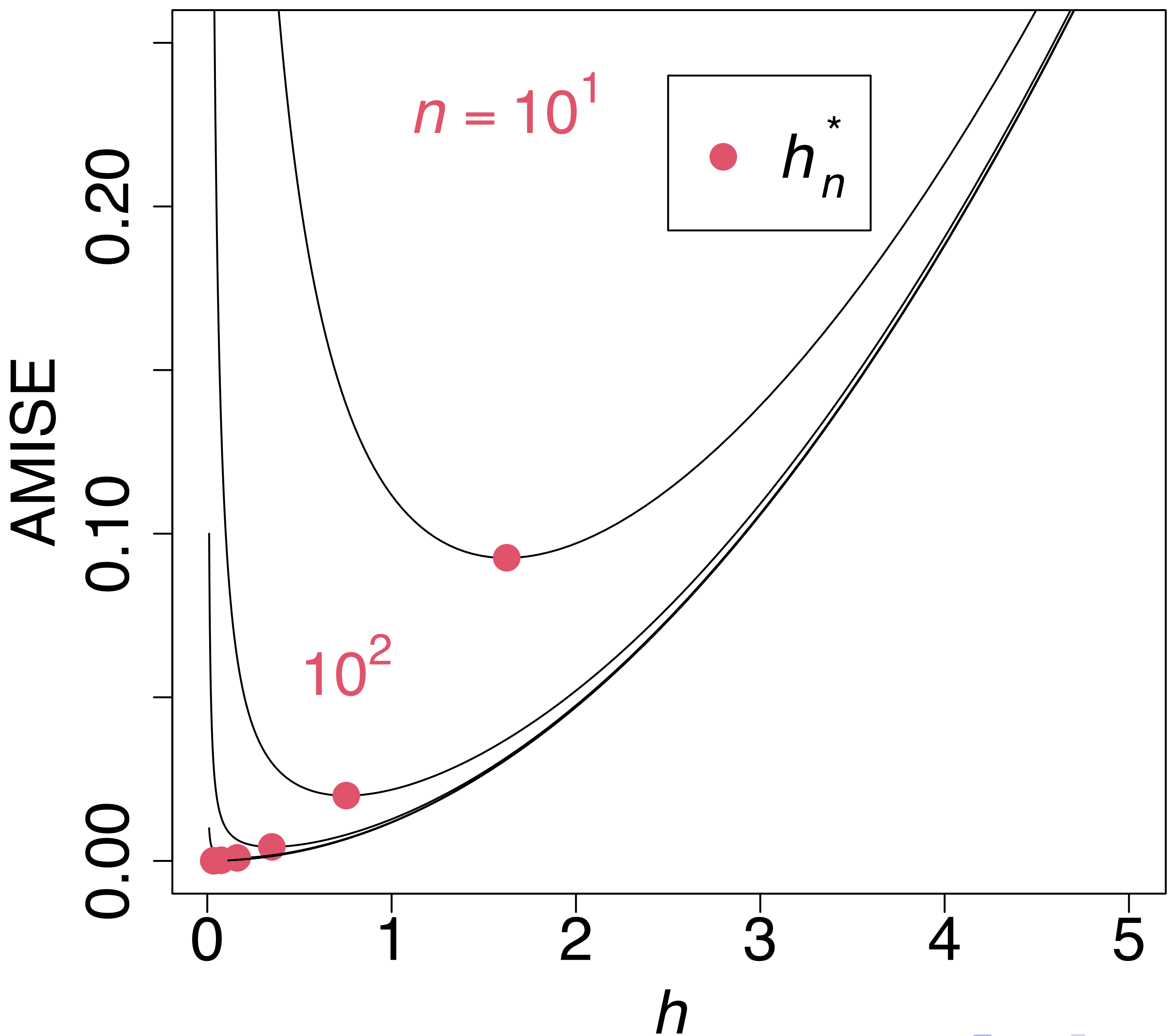
$h \to \infty$

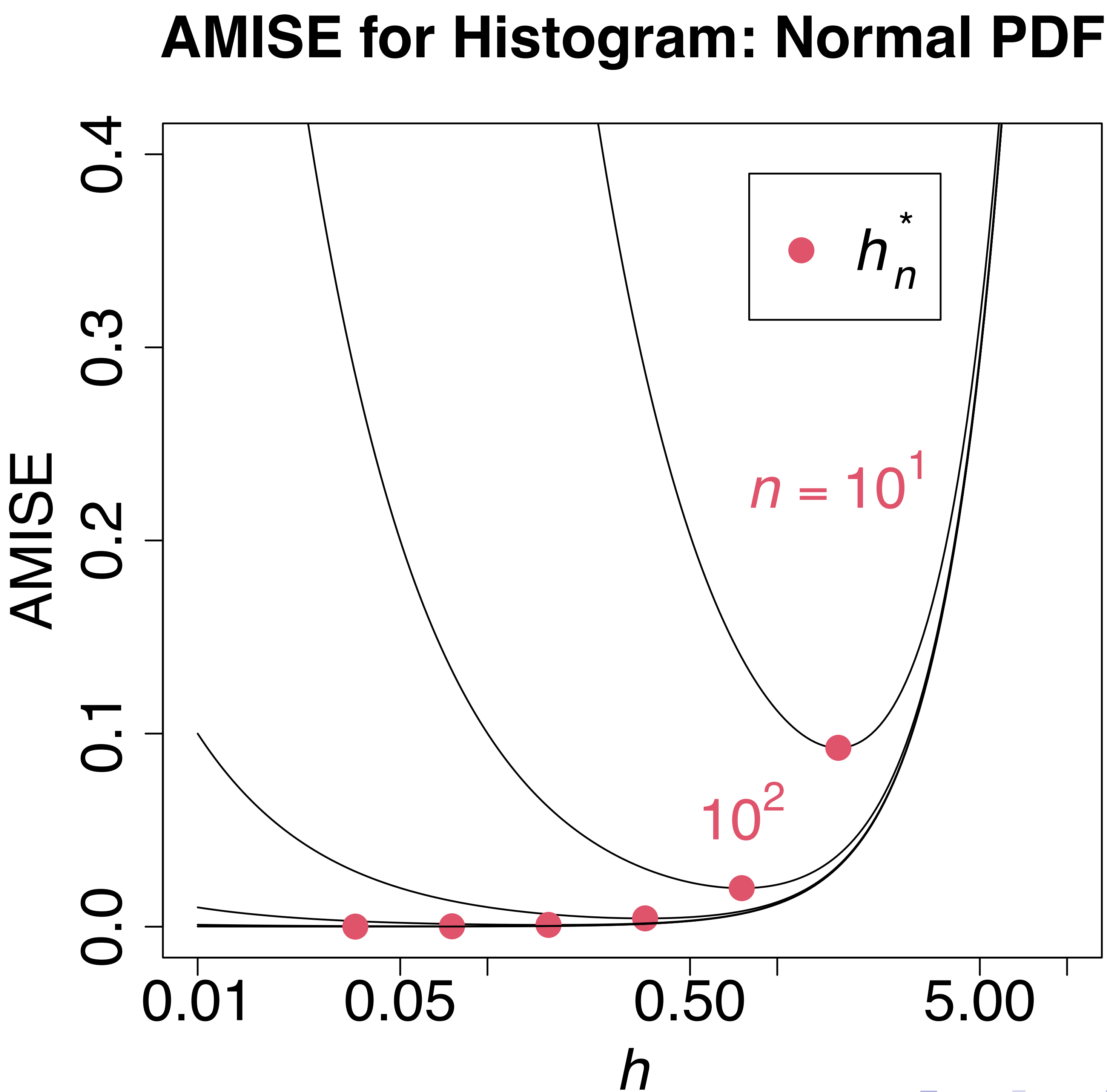
$IQR = 1.35\,\sigma$

$\sigma = \frac{IQR}{1.35}$

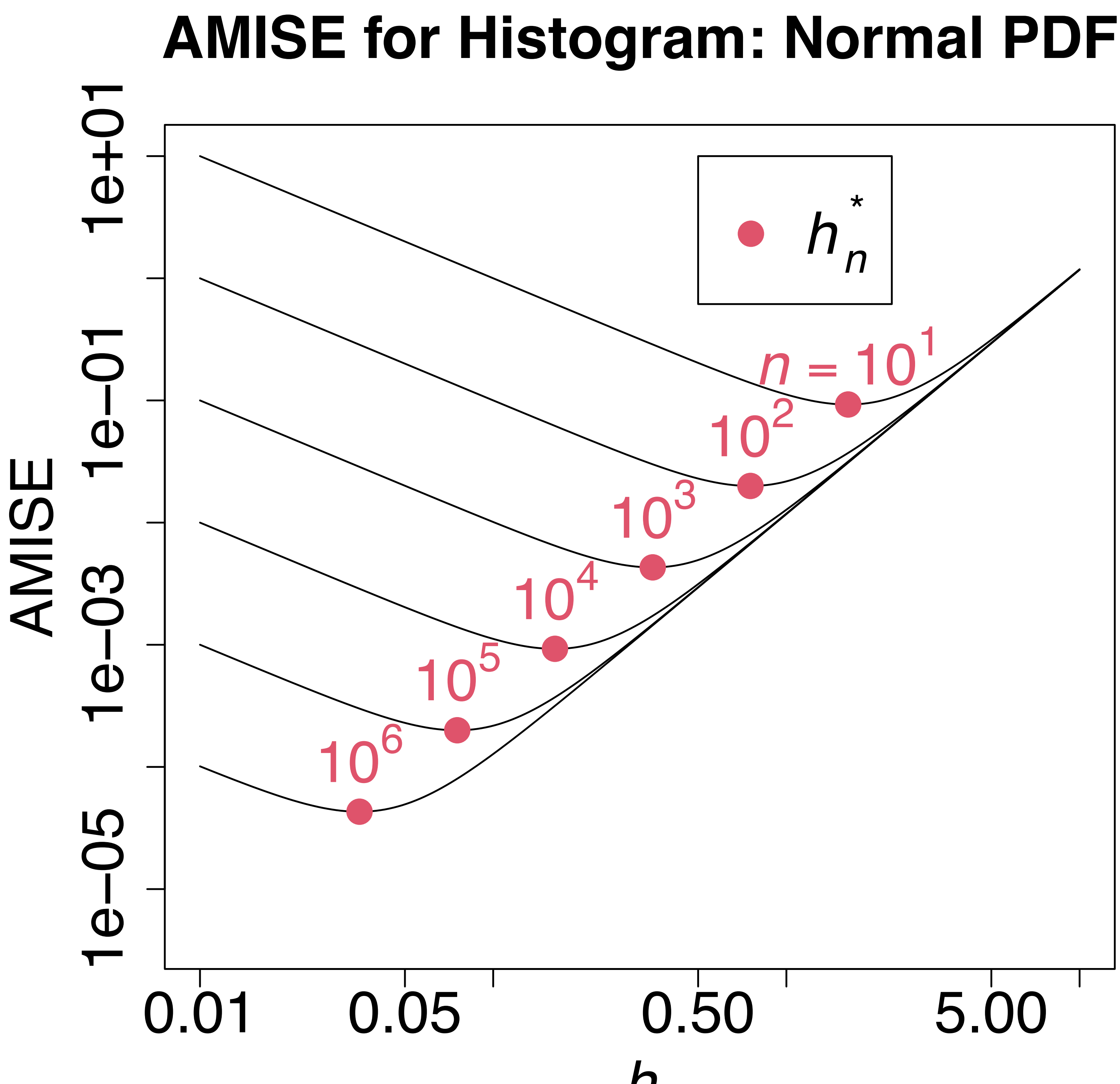
$2.58\, IQR\, n^{-1/3}$

AMISE for Histogram: Normal PDF



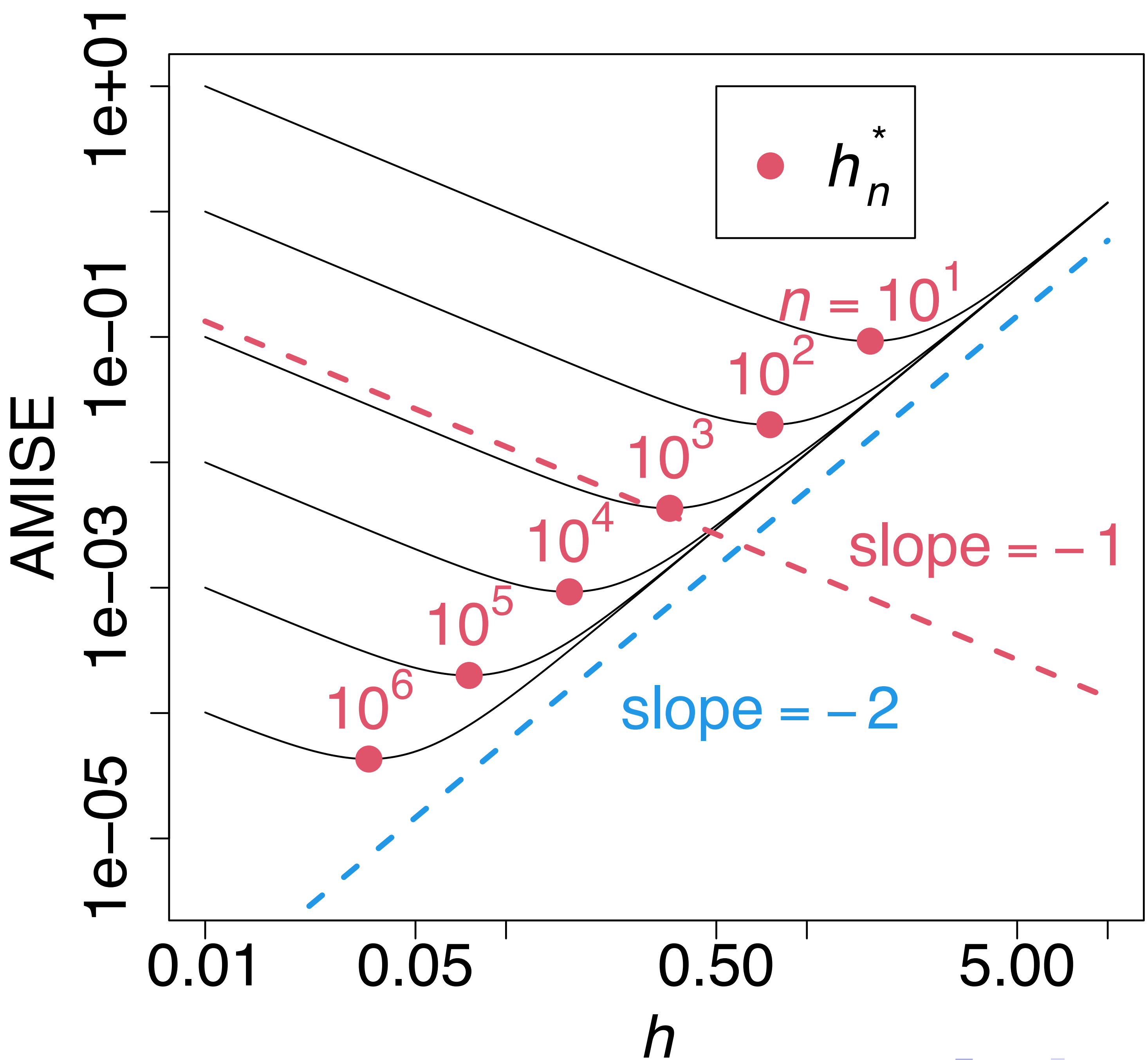


AMISE for Histogram: Normal PDF
AMISE
0.0
0.1
0.2
0.3
0.4
0.01
0.05
0.50
5.00
h
h_n^*
$n = 10^1$
10^2



AMISE for Histogram: Normal PDF
AMISE
1e+01
1e−01
1e−03
1e−05
0.01
0.05
0.50
5.00
h
h*n
n = 10^1
10^2
10^3
10^4
10^5
10^6

AMISE for Histogram: Normal PDF



$$ISE(h) = \frac{1}{n^2 h} \sum \nu_k^2 - \frac{2}{nh} \sum \nu_k p_k + R(g)$$

$$E[\;] = \frac{1}{n^2 h} \sum_k \left\{ n p_k (1 - p_k) + (n p_k)^2 \right\} - \frac{2}{h} \sum p_k^2 + \ldots$$

$$= \frac{1}{nh} \sum_k p_k (1 - p_k) + \sum \frac{p_k^2}{h} - 2 \sum \frac{p_k^2}{h} + R(g)$$

$$= \frac{1}{nh} - \frac{1}{nh} \sum p_k^2 - \frac{1}{h} \sum p_k^2 + R(g)$$

$$p_0 = \int_0^h \left(g(0) + x g'(0) + \frac{1}{2} x^2 g''(0) + \cdots \right)$$

$$= h g(0) + \frac{1}{2} h^2 g'(0) + \frac{1}{6} h^3 g''(0) + \cdots$$

$$\therefore \frac{p_0^2}{h} = h g(0)^2 + \frac{1}{4} h^3 g'(0)^2 + h^2 g(0) g'(0)$$

$$+ \frac{1}{3} h^3 g(0) g''(0) + \cdots + O(h^4)$$

$$\sum_k \frac{p_k^2}{h} \Rightarrow \sum_k g(t_k)^2 \cdot h \quad = R g^2$$

$$+ \frac{1}{4} h^2 R g'^2 \qquad + \frac{1}{3} h^2 \int g(x) g''(x)$$

$$- \frac{1}{12} h^2 R g'^2 \qquad - \frac{1}{3} h^2 R g'^2$$

$$h \int_{-\infty}^{\infty} g(x)\, g'(x)\, dx$$

$$h \, \frac{1}{2} g(x)^2 \Big|_{-\infty}^{\infty}$$

$$\therefore \underline{\text{AMISE}} = \frac{1}{nh} + \frac{1}{12} h^2 R(g')$$

$$-\frac{1}{n} R(g) + \frac{1}{12} h^2 \frac{1}{n} R(g')$$