Stat 550 Virtual Whiteboard

David W Scott

Rice University

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$$AMISE(h, n) = \frac{1}{nh} + \frac{1}{12} h^2 \int f''(x)^2$$

adaptive

$$AAMSE(h) = \frac{f(x)}{nh} + \frac{1}{12} h^2 f''(x)^2$$

$$h^*(x) = \left[\frac{6 f(x)}{n f''(x)^2}\right]^{1/3}$$

$$AAMSE(h^*(x)) = \left(\frac{3 f(x) f''(x)}{4n}\right)^{2/3}$$

$$AAIMSE = \left(\frac{3}{4}\right)^{2/3} \left[\int \{f''(x) f(x)\}^{2/3}\right] n^{-2/3}$$

$$AMISE^* = \left(\frac{3}{4}\right)^{2/3} \left[\int f''(x)^2 dx\right]^{1/3} n^{-2/3}$$

Jensen's Inequality

$$E\left\{\left(\frac{f'(X)}{f(X)}\right)^{2}\right\}^{1/3} \le \left[E\,\frac{f'(X)^{2}}{f(X)}\right]^{1/3}$$

$$\int \frac{f'^{2/3}}{f^{1/3}}\cdot f \qquad \le \qquad R(f')^{1/3}$$

$$\le$$

$$\int (f'\cdot f)^{2/3}$$

oversmoothing on $[0,1]$

$\min \int g''(x)^2$

$$\min \int_{-\infty}^{\infty} g'(x)^2 \, dx \to \infty$$

support $(-1/2, 1/2)$

$\int g = 1$

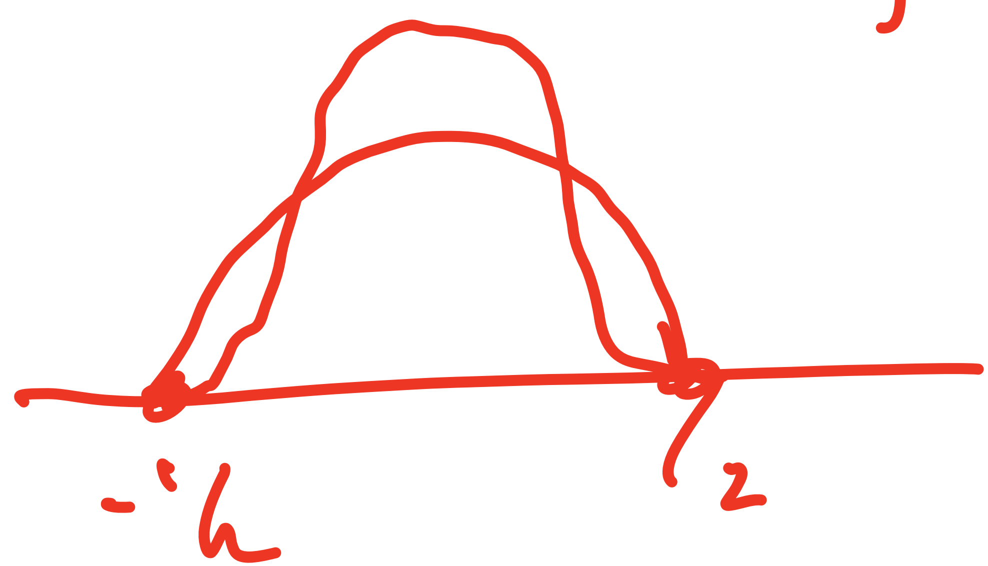
$\delta(t-1/2)$ $\quad \int \delta(t-1/2)^2$

$$\int \delta(t) w(t) \, dt = w(0)$$

$$L(f) = \int_{-\infty}^{\infty} f'(x)^2 \, dx + \lambda \left(1 - \int f(x)\right)$$

$$\frac{L(f+\varepsilon\eta) - L(f)}{\varepsilon} \to 0 \text{ at } f^*$$

at $x = 1/2$



$$= \int (f' + \varepsilon \eta')^2 + \lambda\left(1 - \int f + \varepsilon \eta\right)$$

$$= \int f'^2 + 2\varepsilon \int f' \eta' + \varepsilon^2 \int \eta'^2$$

$$+ \lambda - \lambda \int f + \lambda\varepsilon \int \eta$$

$$\div \varepsilon \quad 2 \int f' \eta' + \varepsilon \int \eta'^2 - \lambda \int \eta$$

$$= 2 \int f'' \eta - \lambda \int \eta = 0 \quad \forall \eta$$

$$= \int \left(2 f''(x) - \lambda \right) \eta(x) = 0$$