Stat 550 Virtual Whiteboard

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$$\min \int_{-\infty}^{\infty} f'(x)^2 \, dx \quad \text{s/t} \quad \int_a^b f = 1$$

(finite support)

$\Rightarrow$ quadratic

$$f^* = \frac{3}{2}\left(1 - 4x^2\right) \qquad a = -1/2, \quad b = 1/2$$

$$R(f^*) = 12/(b-a)^3$$

$$\text{AMISE} = \frac{1}{nh} + \frac{1}{12} h^2 R(f')$$

$$h^* = \left(\frac{6}{n R(f')} \right)^{1/3}$$

$$\le \left(\frac{6}{n \frac{12}{(b-a)^3}} \right)^{1/3} = \frac{b-a}{\sqrt[3]{2n}}$$

$$\# \text{bins} \ge \sqrt[3]{2n}$$

Variation on "prior" kinks

$$\min \int_{-\infty}^{\infty} f'^2 \quad \text{s/t} \quad \int f = 1, \quad \int x f = 0, \quad \int x^2 f = 1$$

$$L(f) = \int f'(x)^2 + \lambda_1 \left[1 - \int f\right] + \lambda_2 \left[0 - \int x f\right] + \lambda_3 \left[1 - \int x^2 f\right]$$

$$L(f + \varepsilon\eta) = L(f) + 2\varepsilon \int f'\eta' + \varepsilon^2 \int \eta'^2 + \lambda_1\left[1 - \int f - \varepsilon\int \eta\right] + \lambda_3\left[1 - \int x^2 f - \varepsilon \int x^2 \eta\right]$$

$-L(f)$

$$2\int f'(x)\eta'(x) - \lambda_1 \int \eta - \lambda_2 \int x^2 \eta = 0 \quad \forall \eta$$

$$2\int f''(x)\eta(x) - \lambda_1 \int \eta - \lambda_2 \int x^2 \eta = \int [2f'' - \lambda_1 - \lambda_2 x^2] \eta$$

$$2f'' - \lambda_1 - \lambda_2 x^2 = 0$$

$$\Rightarrow f'' = \text{~~gerln~~} \quad f^{iv} = \text{~~2 et~~}$$

$$f^* = \frac{15}{16}\left(1-x^2\right)_+^2 \qquad \text{Beta}(3,3)$$