Stat 550 Virtual Whiteboard
Chapter 6 Kernel Density Estimation

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$$\hat{f}(x) = \sum \frac{1}{nh} K\left(\frac{x-x_i}{h}\right) = \sum_{i=1}^{n} K_h(x-x_i)\frac{1}{n}$$

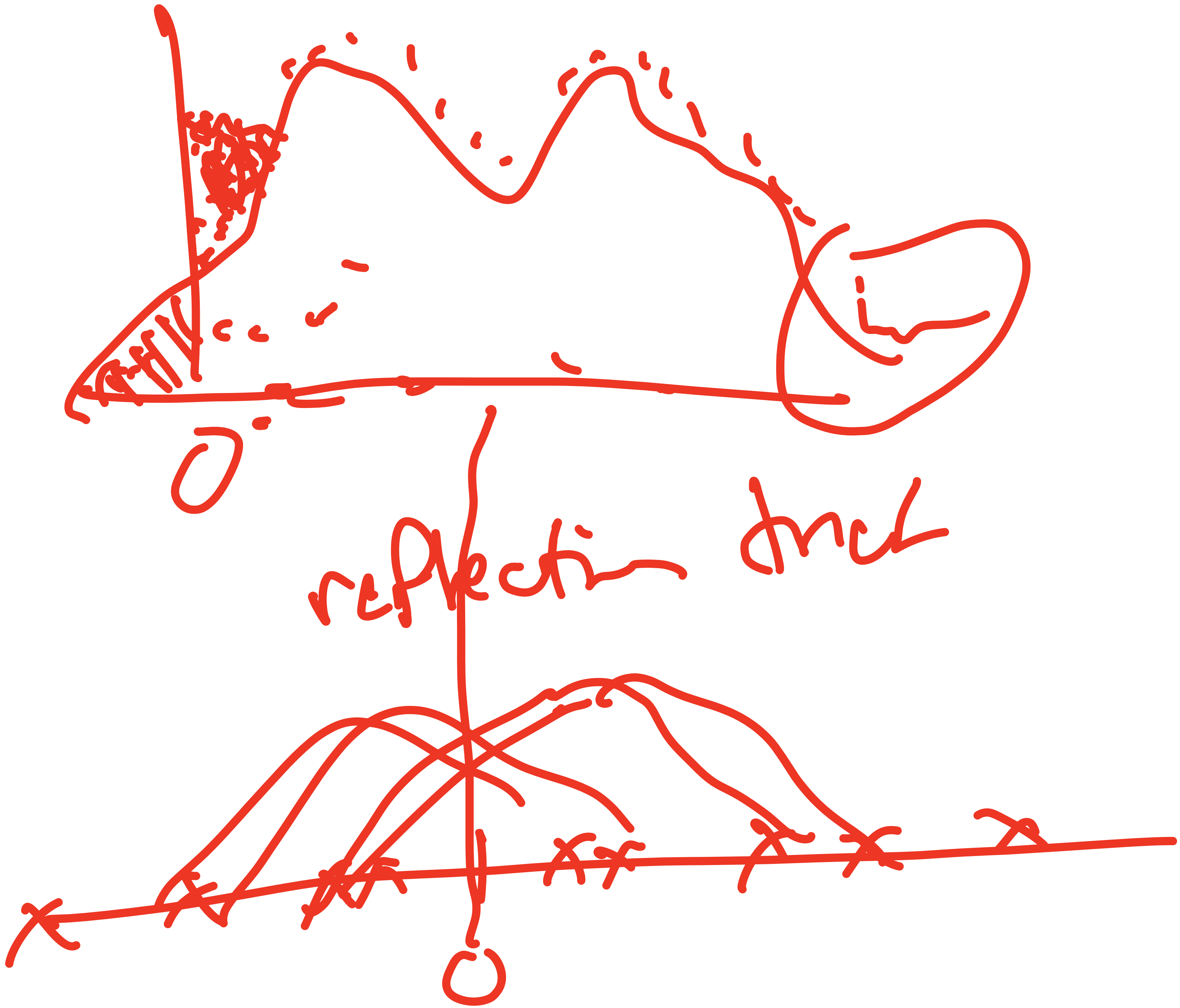
odd powers of $h \equiv 0$

even powers

$$\sigma_K^2 \cdot h^2 f''(x)$$

$$\text{kurtosis} \cdot h^4 f^{iv}(x)$$

$\vdots$


0
reflection trick
0

$\hat{f}(x) = \frac{1}{n} \sum_i K_h(x - x_i)$ works

George Terrell

Thm 6.6 Any estimator of a pdf

$\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K(x, x_i, F_n)$

where K is the Gateaux derivative

$DT(\phi)[\eta] = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \left[T(\phi + \varepsilon\eta) - T(\phi) \right]$

Pf: $F_n(\cdot) = \frac{1}{n}\sum_{i=1}^{n} I_{[x_i,\infty)}(\cdot)$ ←

$\hat{f}(x) = T_n\{F_n\}$

Def $K(x, y, F_n) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\left[T_x\{(1-\varepsilon)F_n + \varepsilon I_{[y,\infty)}\} - (1-\varepsilon)T_n(F_n)\right]$

$= \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\left[T_n\{F_n + \varepsilon(I_{[y,\infty)} - F_n)\} + \varepsilon T_n\{F_n\} - T_n(F_n)\right]$

η

$\hat{f}(x)$

$= \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\left[I_{[y,\infty)} - F_n \right.$

$D T_n(F_n)\left[I_{[y,\infty)} - F_n\right] + \hat{f}(x)$

$$DT_x(F_n)\left[I_{[y,\infty)} - F_n\right] + f(x)$$

$$\frac{1}{n}\sum_{i=1}^{n} K(x, x_i, F_n)$$

$$= \frac{1}{n} D_1 T_n(F_n)\left[\frac{1}{n}\sum I_{[x_i,\infty)} - F_n\right] + f(x)$$

Q: Does this (help) answer our question

A1 $\lim_{n \to \infty} K(x, x_i; F_n) \doteq \delta(x - x_i)$

A2 $\bar{x}$ is local?? **NO**

To get the kernel, perturb F_n

$$(1-\varepsilon)\underbrace{\frac{1}{n}\sum \delta(x-x_i)} + \varepsilon\,\delta(x-y)$$

$$\delta x = (1-\varepsilon)\bar{x} + \varepsilon y$$

<u>ex</u> $f(x) = \phi(x \mid \bar{x}, 1)$

$$K(x,y,F_n) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\left[\phi(x \mid (1-\varepsilon)\bar{x}+\varepsilon y, 1) - (1-\varepsilon)\phi(x \mid \bar{x}, 1)\right]$$