Solution Homework 4 Stat 550 (11-9-23)

1. Problem 5.2 (with typo; see sentence after equation (5.5))

$$\begin{aligned}
\hat{f}(x) &= \frac{1}{nh} \sum_{i=1-m}^{m-1} w_m(i) \nu_{k+i} \\
\int_{-\infty}^{\infty} \hat{f}(x) &= \sum_{k=-\infty}^{\infty} \int_{B_k} \hat{f}(x) dx \\
&= \sum_{k=-\infty}^{\infty} \frac{1}{nh} \sum_{i=1-m}^{m-1} w_m(i) \nu_{k+i} \times \frac{h}{m} \\
&= \frac{1}{nm} \sum_{i=1-m}^{m-1} w_m(i) \sum_{k=-\infty}^{\infty} \nu_{k+i} \\
&= \frac{1}{nm} \sum_{i=1-m}^{m-1} w_m(i) \times n \\
&= \frac{1}{m} \sum_{i=1-m}^{m-1} w_m(i) \\
&= 1 \qquad \text{when weights to } m.
\end{aligned}$$

2. Consider the 2-component normal mixture

$$f(x) = \frac{1}{2}\phi(x|\mu_1, \sigma) + \frac{1}{2}\phi(x|\mu_2, \sigma)\,.$$

How large must $|\mu_1 - \mu_2|$ be so that the mixture is bimodal?

Answer: $\left|\frac{\mu_1 - \mu_2}{\sigma}\right| = 2\,.$

3. Enter the Swiss Franc data into R. (The `read.table` command is in the file.) The data are 6 measurments on the old 1000 franc bills, 100 genuine and 100 counterfeit; see figure below. Which variables seem to suggest the 2 clusters, using the ASH?

The 6th variable is strongly bimodal. See the file `ash.q`