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Nobels for nonsense

Abstract: *We apply exploratory data analysis to some of the basic models of neoclassical computational finance. These include the portfolio selection algorithm of Markowitz, the capital market line of Sharpe, and the option pricing model of Black–Scholes–Merton. We demonstrate that the Markowitzian assumption of positive correlation of expected return and volatility is not supported by the data. The notion that an index fund based on market cap weighting is optimal is also shown to be inconsistent with market data. It is noted that the option pricing model of Black–Scholes–Merton is not supported by market history. The SIMUGRAM™, an empirical data-based paradigm for portfolio selection, is discussed. It is observed that some of the basic contemporary strategies of neoclassical computational finance may be seriously flawed and might profitably be replaced by data-based rules. We conclude that several Nobel Prizes in economics have been awarded for nonsense.*

Key words: *capital market line, efficient frontier, options, S&P 100, SIMUGRAM™.*

Some important decisions influencing the financial well-being of tens of millions of people are based on the neoclassical models associated with the efficient market hypothesis (EMH). The EMH tells us, basically, that markets work quickly to assimilate information so that the true value of a security is essentially the current trading price of that

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0160–3477 / 2006 $9.50 + 0.00.
DOI 10.2753/PKE0160-3477290101

security. If such efficiency were real, then the total market holdings of all investors, based on the market value of each security multiplied by the number of shares outstanding, should be, in a sense, "as good as it gets." If the total market portfolio could be improved on—that is, if we could find another combination of stocks and weights of those stocks that produced a higher expected return for a lower risk—then the total market portfolio, according to the EMH, would immediately adjust itself by price changes to correct this market inefficiency.

Since the beginning of the industrial revolution, a number of economic models have come and gone. Marxist models, which once governed the lives of hundreds of millions, have essentially become extinct except in North Korea and in some American academic departments of history and literature. Even the People's Republic of China, continuing the policies of Deng Xiaoping, has replaced the Marxist model with one difficult to characterize, but something like a much less kind and gentle upscaling of the authoritarian regime of Singapore.

From the late 1930s through the 1970s, the industrial democracies largely followed Keynesian and Post Keynesian modalities. Many of the econometric models, which purported to be Keynesian, were empirical and constantly changing, depending on the fashion of the time (and without the assumption of anything like full efficiency of the market). Keynesian models evolved into unwieldy econometric representations, whose implementations frequently required government interventions of great complexity.

The current age, accurately described as the legacy of figures such as Friedrich Hayek, Milton Friedman, and others, offers holistic models of amazing simplicity. Unlike the Marxist models, which had an essential kernel of fantasy, the neoclassical models, such as the proto version of Adam Smith, make a great deal of appeal to common sense. The current prevailing philosophy forms the basis of modern financial theory. More than half the aspirants for the master of business administration (MBA) degree are specialists in finance, and almost all MBAs are trained in the neoclassical mode.

Critiques of the EMH are relatively rare, and when they are successfully argued, there is a tendency to try and patch the EMH in some clever way so that the objections become obviated. Indeed, because there seems to be a great deal of efficiency in the market, one sees a situation rather like that in the nineteenth century, when Francis Galton found the DeMoivrean (Gaussian, Laplacian) distribution so ubiquitous that he called it *normal.* Deviations from normality, such as imperfections in the emperor's new clothes, could be dealt with, even wished away.

So it has been with the EMH. In part, there has been a failure to carry out checking of some of the consequences of the EMH, because alternatives brought forth by the econometricians have been complex and subject to ad hoc alteration. At the philosophical level, warning flags have been raised concerning the EMH by Bernstein (1996), Findlay et al. (2003), Thompson and Williams (1999–2000), and Williams and Findlay (1974). Modern computing enables us to build empirical data-based paradigms as alternatives to those based on the EMH. If we can find significant discrepancies between consequences of the EMH and the reality of market data, then we should raise warnings for those who believe the world of markets is the efficient evolution of a random walk. Such data stressing of some of the key *theorems* of modern finance, including some whose creators have been awarded Nobel prizes, is the purpose of this paper.

Variance as a surrogate for risk

Given a choice between buying a security with an annual return rate of 0.08 and 0.10 standard deviation or a security with a return rate of 0.08 and a standard deviation of 0.20, most investors will pick the security with the lower standard deviation, ceteris paribus. In Figure 1, we show a typical picture of Harry Markowitz's *efficient frontier.*

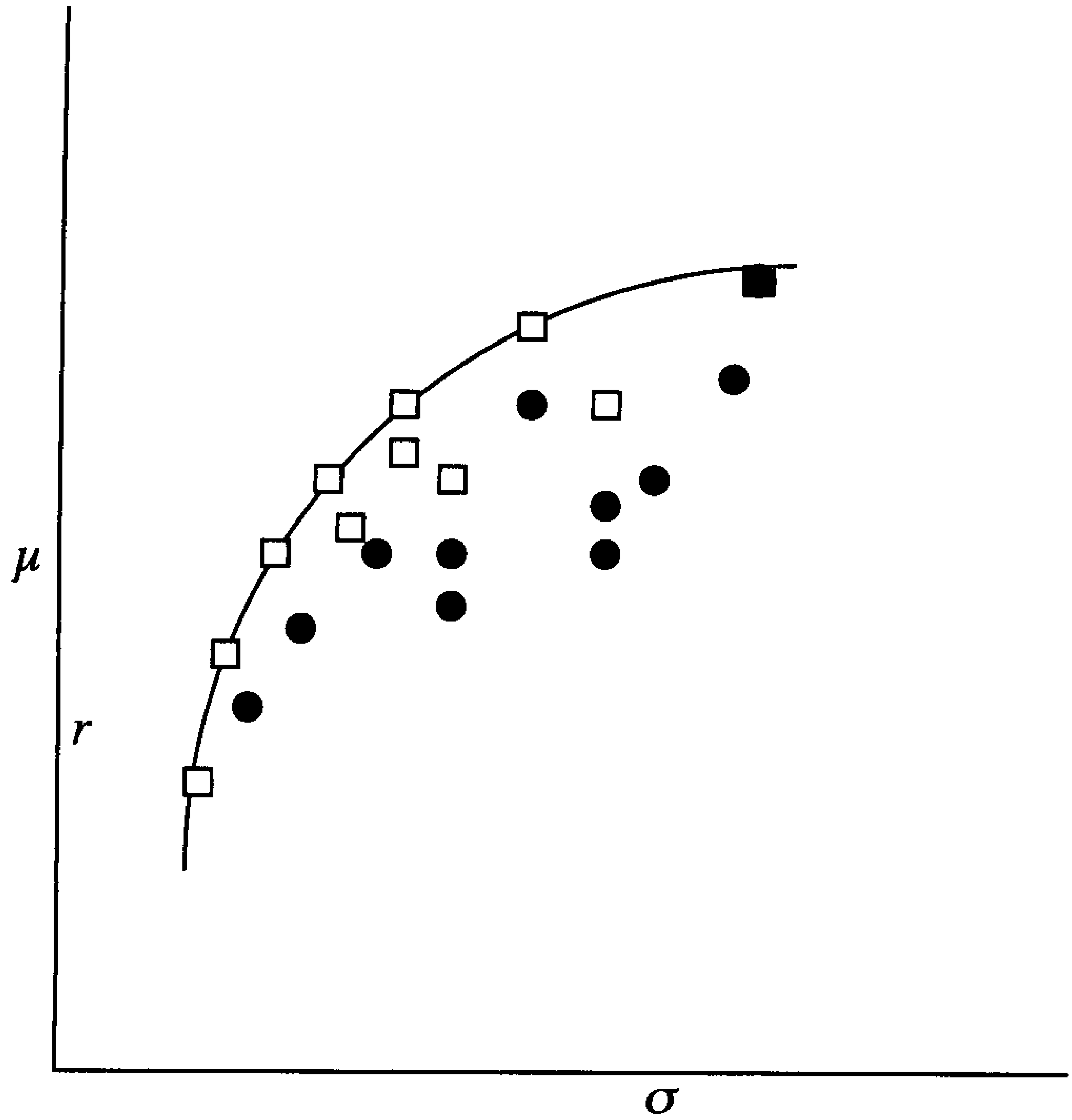
If μ and σ are all that matters, then the securities represented by the black circles should not be purchased. There are other consequences. If a high σ security is ever to be purchased, then it must have a higher μ than lower σ securities. In an efficient market, then, there should be a positive correlation between the expected return of a portfolio and its volatility (standard deviation). The following paraphrase of the portfolio design paradigm of Markowitz (1952; 1956; 1959) thus makes perfect sense:

> Given a set of n stocks and a capital to be invested of C, find the allocation of capital that maximizes the expected return of the portfolio P for an acceptable volatility of the total portfolio σ.

The simple constrained optimization algorithm of Markowitz has become the basis of modern portfolio theory. (There is the old legend that a member of Markowitz's dissertation committee, Friedman, held the view that the result was obviously true but not particularly relevant.) For his work in optimal portfolio design, Markowitz received the Nobel Memorial Prize in economic sciences in 1990.

In Figure 2, we show a σ versus μ plot for 75 years (1926–2000) of one of the oldest large cap indices, the Ibbotson. The correlation is –0.317, negative rather than positive.

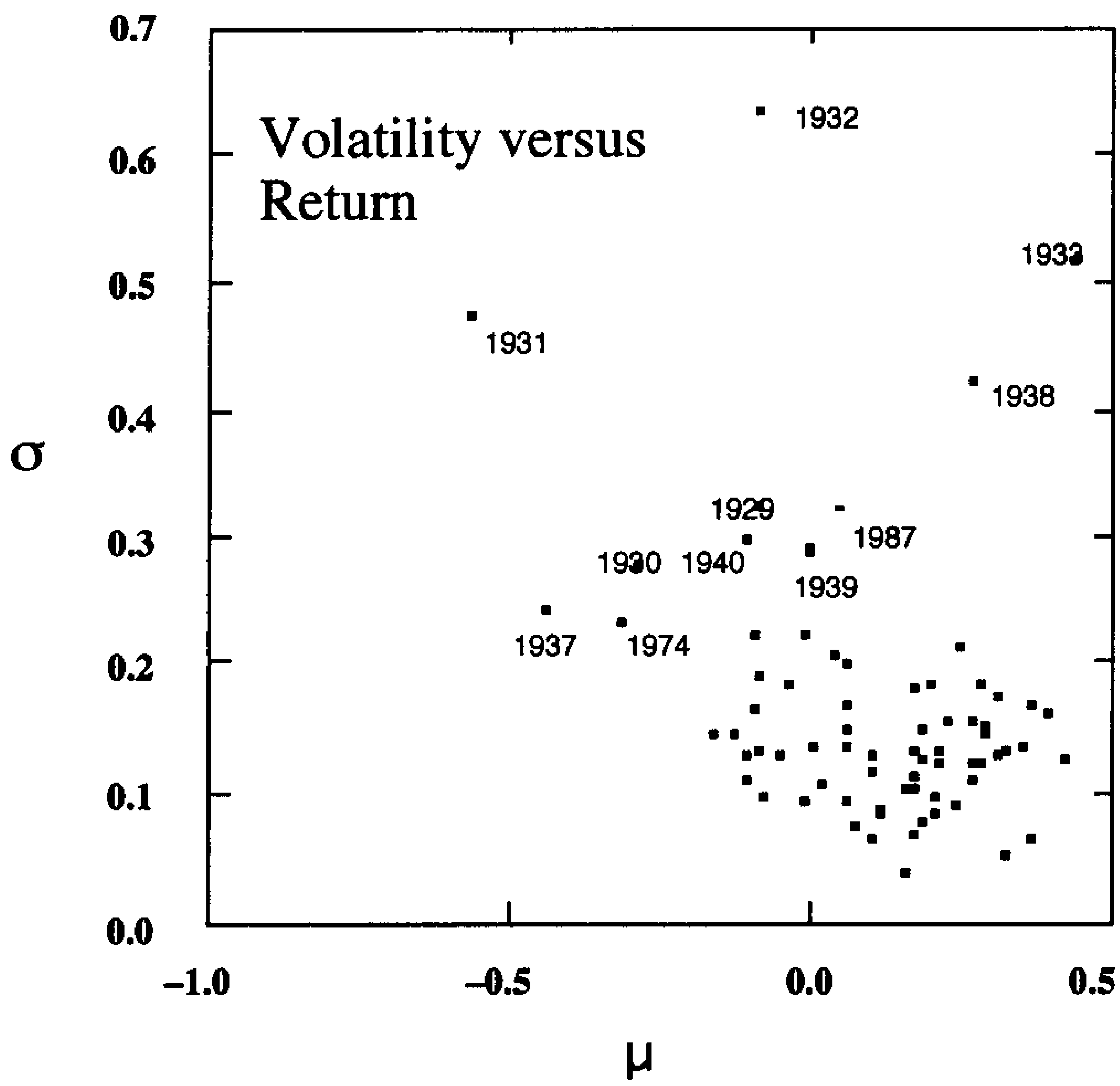
Figure 1 Sketch of an efficient frontier



What are we to make of this? Clearly, the investors, operating rationally in the aggregate as the EMH supposes, see factors in the market beyond μ and σ. If the correlation were approximately equal to zero, we could simply note that Markowitz appears to have assumed a consequence of the EMH that is not substantiated by the historical record. But the correlation is actually negative. We know that a number of financial advisors recommend that investors shift from stocks to bonds in a volatile market. Such advice is not inconsistent with Figure 2. If one had sold off his or her position in an Ibbotson Index fund (such did not always exist formally but were easily created by financial advisors for individual investors) whenever the σ of the Ibbotson climbed above 0.20, much harm could have been avoided.

It is clear that if we restrict ourselves to portfolios having $\sigma < S$, a decreasing upper bound on portfolio volatility, we will obtain ever fewer feasible choices for the portfolio as we decrease S. However, the same would be true if we decided, say, to bound the weighted ages of the CFOs of the companies of the securities considered. In other words, because there is more than a question as to the positive correlation be-

Figure 2 Seventy-five years of Ibbotson index volatility and return



tween return and volatility, the Markowitzian basis of modern portfolio theory is brought into serious question as an empirical matter.

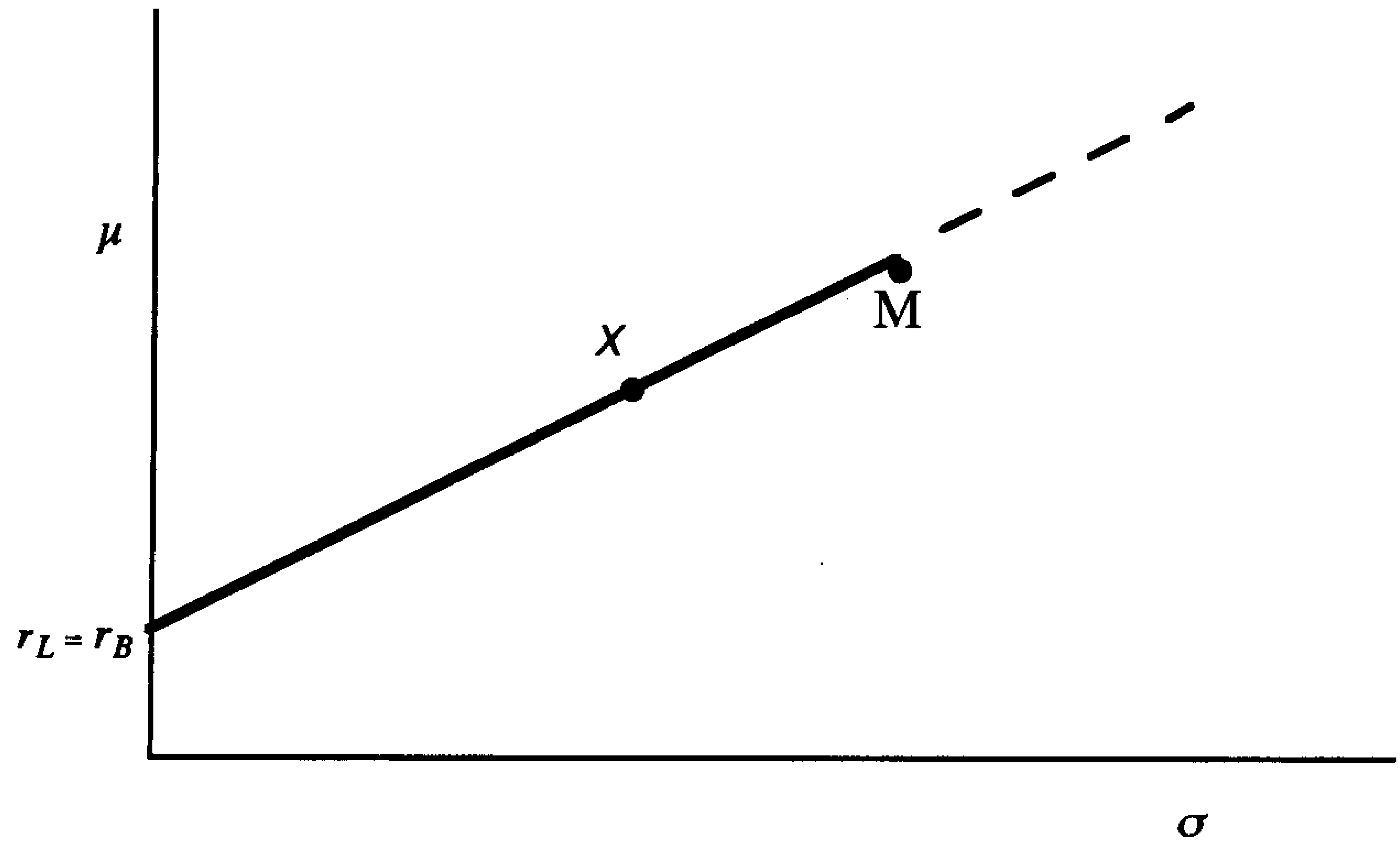
> It is fairly clear that there are more variables associated with a security than μ and σ. Herein lies a serious problem, for, because stock prices do not follow a Gaussian path, μ and σ are insufficient to describe the attributes of a security.

The capital market line and index funds

Let us make the following typical EMH assumptions:

1. the μ and σ of a portfolio adequately describe it for the purpose of investor decision making;
2. investors can borrow and lend as much as they want at the riskless rate of interest;
3. all investors have the same expectations regarding the future, the same portfolios available to them, and the same time horizon; and
4. taxes, transactions costs, inflation, and changes in interest rates may be ignored.

Figure 3 The capital market line



Under the assumptions above, all investors will have identical opportunity sets, borrowing and lending rates ($r = r_B = r_L$), and thus identical optimal borrowing–lending portfolios, say *X* (see Figure 3). Here *M* represents the total market. Because all investors will be seeking to acquire the same portfolio (*M*), and will then borrow or lend to move along the *capital market line* (CML) in Figure 3, it must follow, for equilibrium to be achieved, that all existing securities be contained in the portfolio (*M*). In other words, all securities must be owned by somebody, and any security not initially contained in *X* would drop in price until it did qualify. Therefore, the portfolio held by each individual would be identical to all others and a microcosm of the market, with each security holding bearing the same proportion to the total portfolio as that security's total market value would bear to the total market value of all securities. In no other way could equilibrium be achieved in the capital market under the assumptions stated above.

If an investor wished to go for a higher yield for his or her capital, then he or she could do no better than to borrow at the *r* rate and invest more in the market portfolio *M* continuing on the dashed line in Figure 3. A more conservative investor may choose to have rather more money at the risk-free borrowing rate *r*, winding up at the portfolio *X*. So, he or she would go down the CML toward the riskless rate *r*. A moment's reflection reveals that something like this portfolio is achieved by TIAA-CREF investors, who have a blend of low-risk bonds and a basket of stocks that is an approximation to the total market.

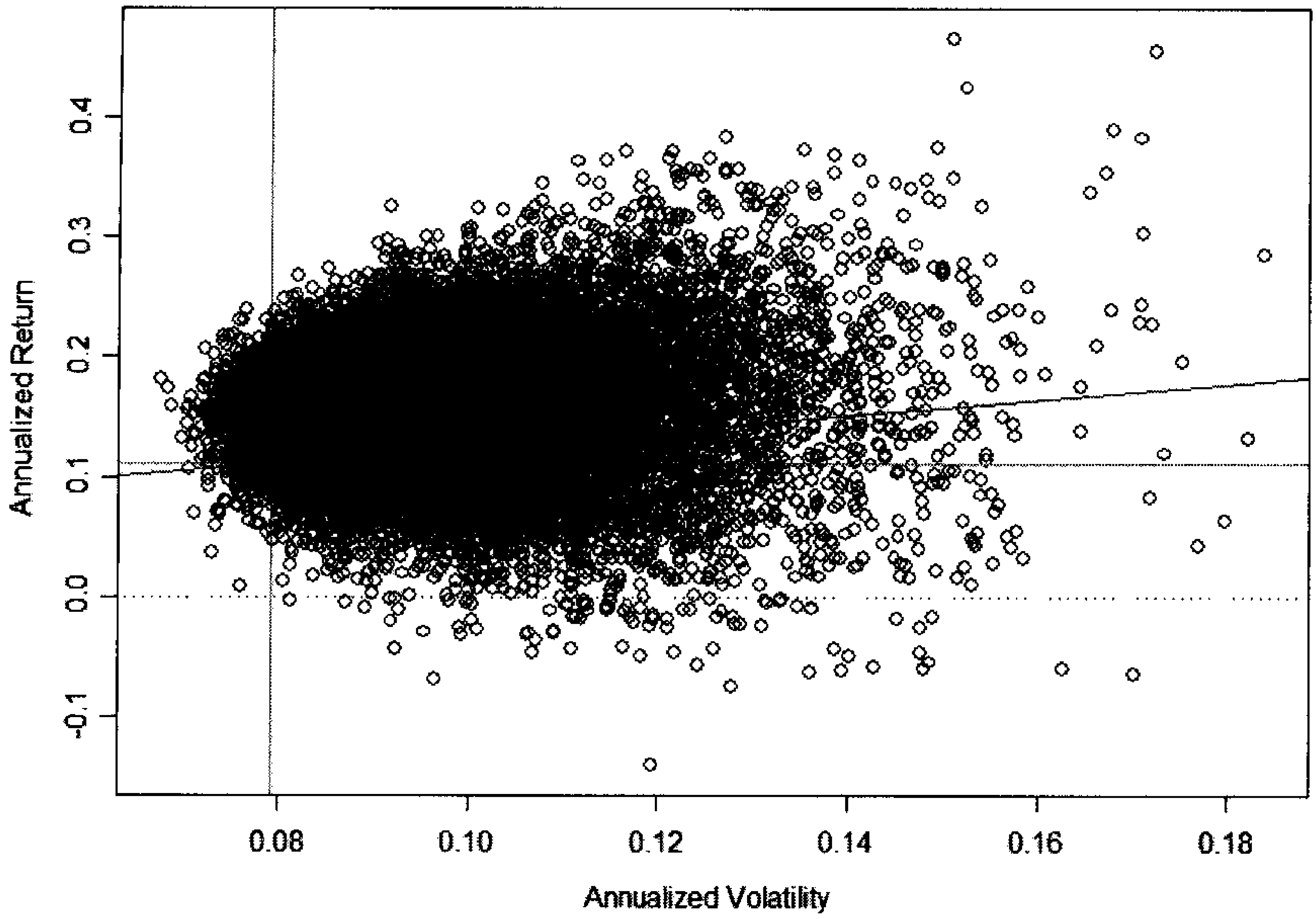
Nobody can do better, based on the axioms above, than pick a portfolio along the CML. Any other strategy must give a portfolio lying below and to the right of the CML. If there were a security that lay above the CML, then it would be such a good deal that it would be purchased quickly, raising its cost (and thus lowering its return) to the point where the end result would lie no higher than the CML. All the portfolios on the CML "superefficient" frontier should dominate those on the Markowitz efficient frontier. This is the theory. And, on the basis of this theory, we have a plethora of market cap weighted index funds, ranging from full market funds based on the Wilshire, to "spider" funds based on a combination of securities within a specific sector. For his development of capital market theory, William Sharpe (1962; 1964; 1972) received, in 1990, the Nobel Memorial Prize in economic sciences.

The work of Sharpe may be said to form the basis of index funds. From this standpoint, all investors should hold the market portfolio leveraged or deleveraged by moving up and down the CML to achieve whatever level of volatility they can tolerate. It should not be possible to create a portfolio lying above the CML. According to the theory of some, such as John Bogle (1999), founder of the Vanguard Funds, one should consider simply investing in a basket of funds that emulates the total market. There is little doubt that, over the past several decades, index funds have tended to best the managed funds. True believers in the EMH might hold that this result is the clear consequence of the efficiency of the market.

Recently, Wojciechowski and Thompson (2006) have considered looking, year by year, at 50,000 portfolios consisting of random selections of stocks from the 1,000 highest market cap securities. We note the results for 1993.

For the 33 years from 1970 through 2002, not simply a flukish few, but a staggering 65 percent of the portfolios selected randomly from the 1,000 largest market cap stocks lie above the CML. So, now we have empirical evidence to the effect that index funds do not really have some sort of cosmic connection to optimality. The aggregate consequence of the 33 plots shown in Figure 4 for the years 1970–2002 indicates that the reason the managed funds have done so poorly is that they have not generally been well managed and most have substantial management fees. If we can beat the index with a randomly selected portfolio, that does not indicate that we should use random selection as our new evangel for portfolio design. Rather, we have simply put to rest the notion of the intrinsic optimality of index funds. We now know that there is hope for market analysis that swims against the tide of Markowitz, Sharpe,

Figure 4 Random portfolios and the capital market line for 1993



and the EMH more generally. In a later part of this paper, we show some results for a technically designed portfolio based on the SIMUGRAM™.

Black–Scholes–Merton and their amazing money machine

We recall that a *call option* is the right (but not the obligation) to buy a security of current price $S(t)$ for strike price X at a future time T.

At this point, we note that there are situations where options may be a good thing. A corn farmer with a healthy crop in view may decide to realize needed capital two months before harvest by selling options on some or all of his or her crop. This is the "bright side of the force."

The "dark side of the force" has to do with using options as a means to get around the margin rules put in place by the Securities and Exchange Act of 1934. Before this Act, one could leverage new purchases of securities against his or her existing portfolio using a margin ratio as high as ten-to-one. That meant that if one had a portfolio consisting of 1,000 shares of a stock selling for $100, one could buy an additional 9,000 shares of stock at $100 per share. However, if the price of the stock then dropped below $90, the broker would seize the investor's entire holding to cover the "margin call." Obviously, in a falling market, such flexibility of margin could cause an unstable control situation where stocks spiraled downward. In the 1930s, the rules were changed so that margin ratios could not exceed two-to-one.

Using options as a way to exercise leverage would appear to get around this difficulty of unstable control, because the option is purchased up front. However, let us consider a company that sells electricity options to local utilities based on uncovered obligations (for example, the company does not own gas or oil assets that can be used to generate the electricity). Then, suppose prices rise continually over the time before the option can be exercised. The company might simply have to default on its option obligations, because it has overreached its resources. Such a default jolts the system, for then the local utilities have to go back into the spot market to obtain electricity. This forces contract prices to go up, and we are back to a state of unstable control. For this reason, Warren Buffett has called options "financial weapons of mass destruction."

What is the "fair price" C of a call option? Of course, the answer is "there is no such thing." However, if we insist on finding such a price based on expectation, one natural answer is

$$\begin{aligned} C &= \exp(-\mu T)E\{\max(0,S(T)-X)\} \\ &= e^{-\mu T}\left(e^{\mu T}S(0)\Phi\left(\frac{\log(S(0)/X)+(\mu+\sigma^2/2)T}{\sigma\sqrt{T}}\right)\right) \\ &= X\Phi\left(\frac{\log(S(0)/X)+(\mu-\sigma^2/2)T}{\sigma\sqrt{T}}\right). \end{aligned} \tag{1}$$

Here, we have brought the price to present value, using the rate of the security.

For reasons too convoluted and political to go into here, the investment banking industry in the early 1970s wished to come up with an answer that did not involve the parameter μ. μ is, after all, an unknown that has to be estimated from past data plus whatever additional information can be gleaned. If we could remove μ from the picture and replace it with a more or less known interest rate r, that would be preferred. Of course, then we would have taken a noisy phenomenon, such as the market price of a stock at a future time, stochastized it further by looking at a derivative value of that price, and come up with a deterministic evaluation. One might suppose that the tailors of the emperor's new clothes would have to be very artful indeed to come up with such a result. In 1973, Black and Scholes produced such a garment:

$$C_{BS} = e^{-rT}\left(e^{rT}S(0)\Phi\left(\frac{\log(S(0)/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right)\right.$$
$$\left. - X\Phi\left(\frac{\log(S(0)/X)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right)\right). \quad (2)$$

μ has been eliminated, because it is now assumed that we continuously rebalance a covering portfolio at zero transaction costs. In 1997, for their work, Scholes and Merton (Black having passed away) received the Nobel Memorial Prize in economic sciences.

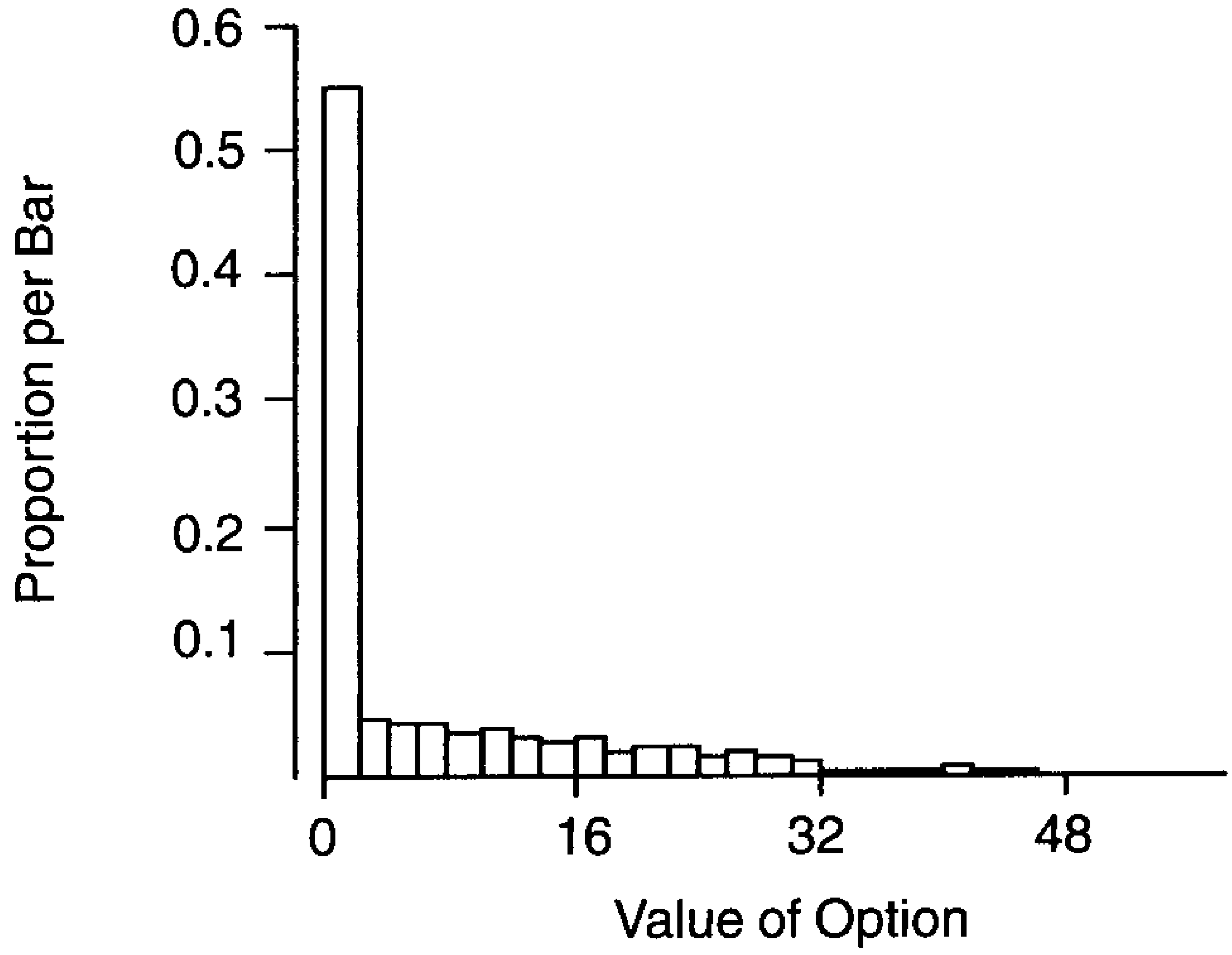
The Black–Scholes price might be said to be "politically correct" from the standpoint of modern neoclassical finance. The Black–Scholes result is shaken in the face of the unwary with the same intensity used by the followers of Mao Zedong with their little red book during the Chinese cultural revolution.

Let us simply note, in passing, that transaction costs really are not free, and that continuous rebalancing is hard to approximate economically. But, beyond that, looking at historical data, we can observe that the Black–Scholes price for an option seldom agrees with the actual price quoted on the ticker. There is, apparently, a neat way around this dilemma. One does not use a historically data-based estimate for σ. Rather, we read from the ticker the actual market price of the option, plug it into Equation (2), and solve for the "implied volatility" σ. This kind of strong circularity of argument would seem to insulate the Black–Scholes result from criticism based on real data. However, such is not the case. *For two different strike prices (X) with the same date of execution, we typically get two different values of implied volatility.*

The Black–Scholes result, applied to all manner of exotic derivatives, would appear to provide hope for employment for those who like to develop numerical approximations for solving the classical heat equation of mathematical physics. However, the difficulties experienced by Enron and other companies that relied strongly on the Black–Scholes result to run their businesses must begin to cast doubt on the model.

Even if the Black–Scholes engine were not hopelessly flawed, from the standpoint of the investor, we note that looking at expectations and variances does not tell the whole story. We need to look at the time

Figure 5 Finishing "out of the money"



indexed distribution functions of the payoffs. Consider the case of a buyer considering the purchase of options on a stock with initial price of \$100. Suppose he or she correctly guesses that $\sigma = 0.20$. Suppose the Treasury bill interest rate is $r = 0.05$. Suppose T is six months and the strike price $X = \$108$. Suppose that $\mu = 0.15$. The Black–Scholes price for the option is \$3.54. The expected value of the option is \$7.23. We show in Figure 5 that in this "best of all possible worlds" scenario, the investor has bought an option that is of no value 55 percent of the time. Once again, we note that building strategies based on expectations is dangerous. And we further observe how quickly we can buy into someone else's use of misleading language. A "hedge" implies a kind of conservatism. Yet "hedge funds" frequently are built on the buying and selling of options. So, accepting the notion that a "hedge fund" is safe is rather like accepting a sign reading "Honest John the Used Car Salesman" at face value.

When models fail

When an individual investor acts on bad information, bad things can happen to his or her portfolio. When powerful officials insist on following bad models, the data notwithstanding, bad things can happen to the economy of the nation. In 1998, the chairman of the Federal Reserve, Alan Greenspan, organized a \$3.5-billion bailout of the failed Long-Term Capital Management (LTCM) "hedge fund." LTCM, like Enron, produced nothing. It simply bought and sold stocks, bonds, and derivatives with leveraging aplenty (typically, a "hedge fund" is a collection of speculative ventures). It was organized based on the "risk neutral" theories of Black, Scholes, and Merton. Indeed, Scholes and Merton were conspicuous advisors to LTCM. It is significant that Greenspan did not question the underlying models followed by LTCM. Rather, he worked on the assumption that the LTCM failure was an extremely rare event unlikely to repeat itself.

Unfortunately, Greenspan acted like a true believer who, when facts are not in accord with cherished beliefs, fails to use facts to modify theory. He reacted quickly to avert the embarrassment caused by what was supposed to be a "six sigma event." Across America, company chieftains, growing accustomed to cooking their books in order to gain the time necessary for their "risk neutral" approaches to bear fruit, heaved a collective sigh of relief and redoubled their cooking. The large accounting firms, whose external reviews were supposed to prevent sharp practice, proceeded to take ever more lenient views of the cunning of their clients. Indeed, the writing of uncovered options and other dubious business practices expanded after LTCM.

From the standpoint of the dollars involved, the 1998 crash of LTCM was orders of magnitude less significant than that of the $62-billion Enron debacle in late 2001. The Enron collapse was too large for even Chairman Greenspan to make disappear. Then, there is the long list of other companies zapped by belated discovery of their irresponsible accounting practices in 2002 and subsequently. The total wreckage will easily top 100 times the LTCM figure.

Even more important, there is the long list of the high-tech companies destroyed by Greenspan's cutting off of investment capital beginning in 1999 (and continuing for a punishing 18 months) as a means of damping down "irrational exuberance." Had nature been allowed to take its course with LTCM in 1998, it is likely that a general scrutiny of accounting practices would have precluded the devastating crash of Enron in 2001. Chairman Greenspan could have seen a natural dampening of "irrational exuberance" already in 1998 had he simply let LTCM fail. By bailing out LTCM, on one hand, and stifling investment capital, on the other, it appears he acted with the wisdom one tends to associate with the economic planners of, say, Argentina.

Persons on fixed incomes must now live with low bank interest rates after Greenspan tried to resuscitate the hypoxic stock market (which he had deprived of oxygen for 18 months) by dropping interest rates over 500 basis points from the highs in 2000. Thus, Greenspan has dealt the retired a double blow: first by undercutting the values of stocks by raising interest rates and then by slashing interest rates to ridiculously low levels so that what little money was left earned very meager returns. One may pity the elderly gentleman who retired in 2000, only to see the value of his stock portfolio cut in half. One such fellow, actually known by one of the authors, described his 401(k) plan as having become a "201(k) plan." Nervous about real risk (not the Markowitz variety), this fellow sold out from fear only to reinvest in short-term CDs, which soon yielded next to nothing. He now greets customers at a leading discount department store.

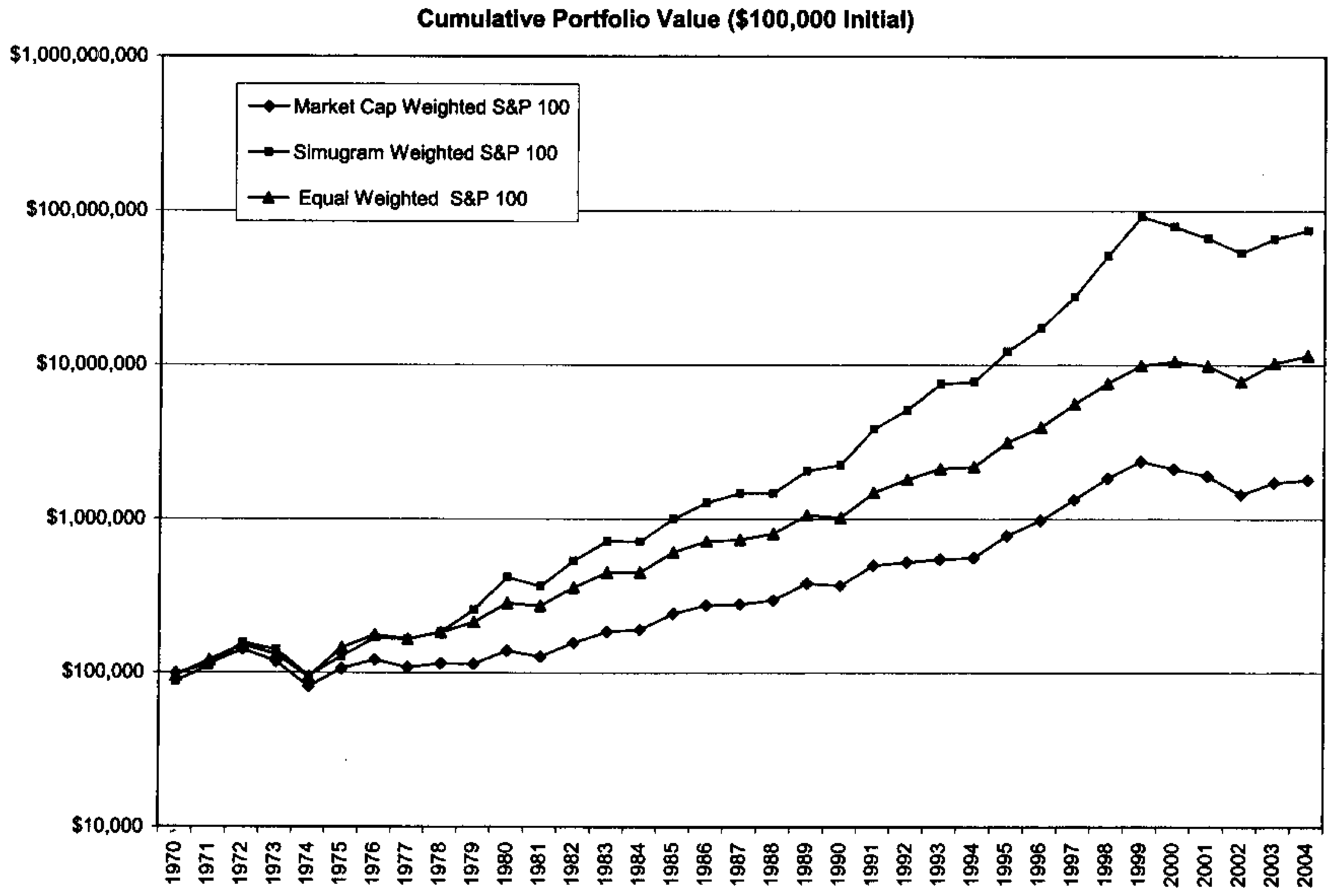
At this point, we should recall Tukey's maxim: "Far better an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise" (1962, p. 13).

Empirical data-based strategies

Some of the elegant technical strategies based on the EMH seem to lack data-supported validity. That need not mean that all is lost. On the contrary, we have the opportunity to build new technical strategies of our own. Because we have no model to bring forth as an alternative to the EMH, we suggest looking at empirical data-based strategies. One strategy that we shall not propose is that of a portfolio based on random weights on stocks. We have seen that market cap weighted index funds may be worse than those obtained by random weights. That does not show that randomized strategies are good; it shows that index funds are not good.

One thing we might try is to build an "equal weight fund" with equal amounts of capital invested in each security in the portfolio. (Perhaps the weighting by market cap penalizes the portfolio for investing too much in large companies.) In Figure 6, we show what would have happened had we used an equal weighting strategy using stocks from the S&P 100, rebalancing the portfolio once a year, during the years 1970–2002. The aggregate return is equivalent to a continuously compounded interest rate of 13.2 percent. This compares to an S&P 100 return of 8.4 percent. (Both returns are exclusive of dividends.) It is also to be noted that over the 32 years, the total negative returns in losing years are –90.57 percent for the equal weight portfolio, as opposed to –118.13 percent for the S&P 100 index fund. We have no immediate explanation for the superior performance of the equal weight portfolio. There are, as we have seen, excellent theoretical explanations as to why the S&P 100

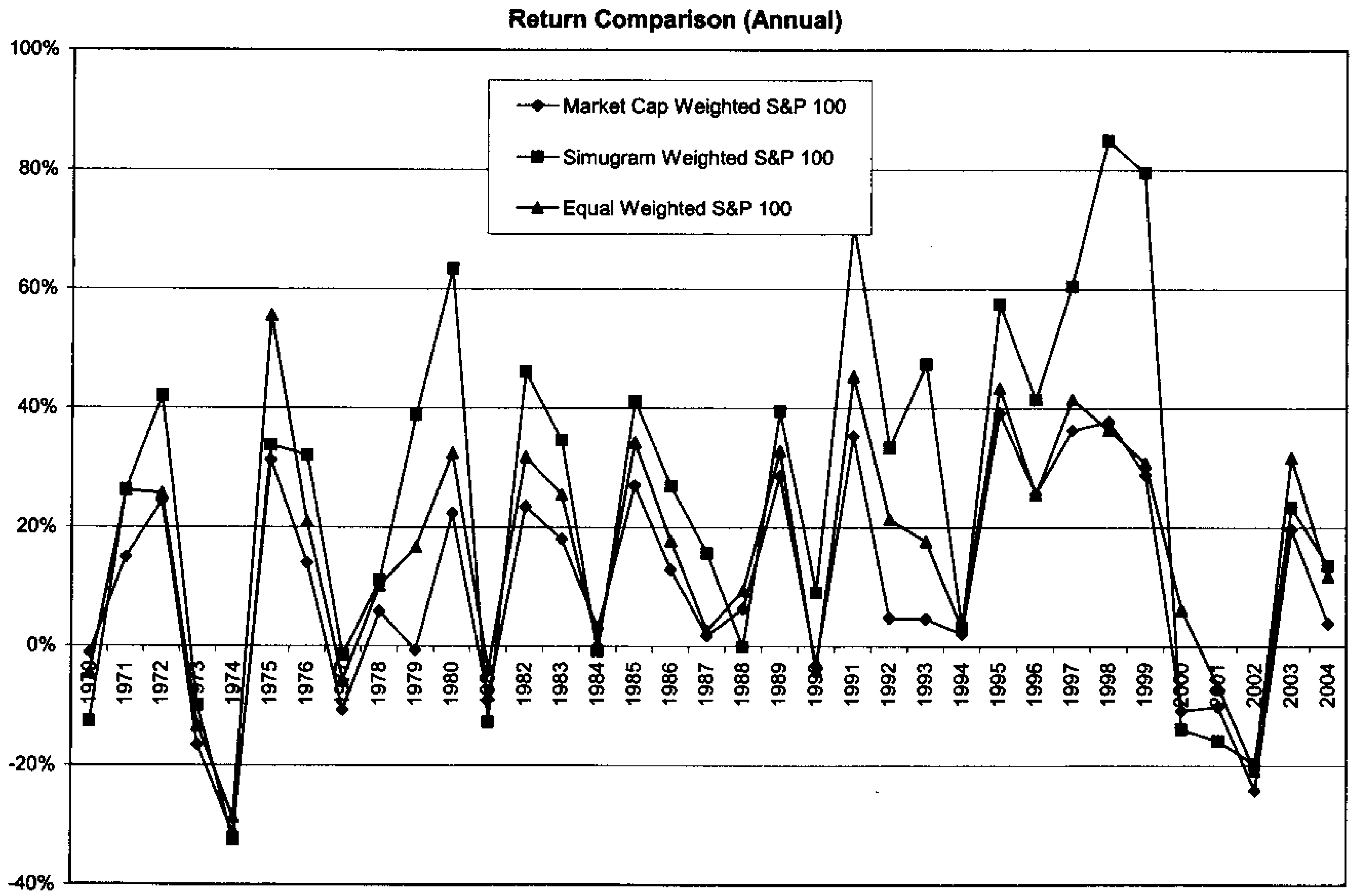
Figure 6 An aggregate comparison of portfolio strategies



should be superior to otherly weighted funds. Unfortunately, here theory does not conform to facts. Our general recommendation when theory does not conform to facts is to try to develop, ultimately, an alternative theory. For the short to medium term, we should develop strategies based on rules developed empirically from data.

Other strategies might be based on rules much more complicated than that of equal weighting. One such is the patented SIMUGRAM^{TM} portfolio paradigm of Simugram Systems Inc. (For a prototype version, see Thompson et al., 2003.) This is a computer-intensive expert system that looks at the synchronized historical performance of the stocks in a selection set and uses this information to develop a high-return, low-risk portfolio. Again, the portfolio is rebalanced once a year. (The SIMUGRAM^{TM} algorithm does not generally conform to a "buy and hold" strategy.) We show the results of 35 years of applying the SIMUGRAM^{TM} portfolio paradigm, ex ante, to the stocks in the S&P 100 in Figure 6. We note that the aggregate return, exclusive of dividends, is equivalent to a continuously compounded interest rate of 20 percent. This is a rate of return comparable to those generally associated with Buffett's Berkshire Hathaway. From Figure 7, we note that over the 35 years beginning in 1970, the total negative returns in losing years are

Figure 7 Year-by-year comparison of portfolio strategies



−112.74 percent, less than those experienced by the S&P 100 but more than those of the equally weighted portfolio.

Conclusions

Three Nobel Prize–winning results in computational finance have been stressed by data and found wanting. New questions have been raised as to the value of structures resting on the foundations of the EMH. The use of flawed models by true believers can cause mischief not only for individual investors but also for the economy generally. The old maxim of Samuel Pepys, "[b]e most slow to believe what you most believe to be true," still holds.

We have demonstrated, however, that there are possibilities for building investment strategies that are dependent not on models but on data-based empiricism. As Vilfredo Pareto taught us a century ago, there is an underlying current of rational order in economics. But the world is a complex, dynamic system of systems consisting, in large measure, of departures from simplistic "laws" seemingly complete, but in reality, incomplete. Our task should be to maneuver in this real world of staggering complexity. To assist us in this task, we have modern high-speed computing coupled with speedy access to huge bases of data. Thomas Alva Edison, in his quest for producing a usable electric lightbulb, tried hundreds of filaments oriented toward the goal of always moving toward better candidates. It can be argued that this was engineering rather than science. Just so. We are in an age in computational finance where our efforts should be directed toward an engineering approach. Our software algorithms are the filaments and computers and databases our laboratory.

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