1.4

(a) B_1

(b) $B_1^c \cap B_2 \cap B_3$

(c) $\bigcap_{1}^{4} B_i$

(d) $\bigcup_{i=1}^{4} (B_i^c \cap (\bigcap_{\substack{1 \le j \le 5 \\ j \ne i}} B_j))$

(e) $B_1 \cap B_2 \cap B_3 \cap B_4^c \cap B_5^c \cap B_6^c \cap B_7^c$

1.11

$$P[total \ge 100] \le P[at\,least\,one \ge 10] \le 1 - P[all < 10] = 1 - (1 - 0.01)^{10} = 0.096 < 0.1$$

or

$$P[total \ge 100] \le P[at\,least\,one \ge 10] = \sum_{1}^{10} P[ith \ge 10] = 10 \times 0.01 = 0.1$$

or

$$P[total \ge 100] \le P[1st \ge 10, rest < 10] + P[2nd \ge 10, rest < 10] + ... + P[10th \ge 10, rest < 10] < 0.1$$

1.20

(a) $\frac{4!}{2!2!} = 6, \frac{4!}{2!1!1!} = 12$

(b) If $n_A = ... = n_Z = 1$, then $possible\# = n!$

If $n_A \ne 1$, then we have $n_A!$ repetition, $possible\# = \frac{n!}{n_A!}$

Therefore, for any $n_A, ..., n_Z$, $possible\# = \frac{n!}{n_A! \cdots n_Z!}$

1.32

(a)

$$P[no\,match] = 1 - P[at\,least\,1\,match] + P[at\,least\,2\,matches] - ...P[at\,least\,n\,matches]$$

$$= 1 - \frac{(n-1)!}{n!} \times \binom{n}{n-1} + \frac{(n-2)!}{n!} \times \binom{n}{n-2} - ... \frac{0!}{n!} \times \binom{n}{n}$$

$$= \sum_{0}^{n} \frac{(-1)^j}{j!}$$

$$so, n_0 = P[no\,match] \times n! = n! \sum_{0}^{n} \frac{(-1)^j}{j!}$$

(b)

$$n_j = \binom{n}{n-j} \times (n-j)_0 = \frac{n!}{j!}\sum_0^{n-j}\frac{(-1)^i}{i!}$$

$$so, P[j\, matches] = \frac{n_j}{n!} = \frac{1}{j!}\sum_0^{n-j}\frac{(-1)^i}{i!}$$

$$\lim_{n\to\infty} P[j\, matches] = \frac{1}{j!e}$$

1.38

$$P[A] = \frac{a}{n}, P[B] = \frac{b}{n}, P[A\cap B] = \frac{c}{n}, where\, a < n, b < n, c < a \wedge b, i.e. c < \min(a,b)$$

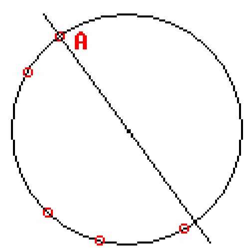
$$If\ P[A\cap B] = P[A]P[B], then, c\times n = a\times b$$

$$That's\ not\ possible\ for\ n\ prime\ and\ a,b,c \text{ int}$$

1.54

Truth table:

A	B	C	D	E	P(E)
1	1	1	1	1	1/81
1	1	1	0	0	2/81
1	1	0	1	0	2/81
1	1	0	0	1	4/81
1	0	1	1	0	2/81
1	0	1	0	1	4/81
1	0	0	1	1	4/81
1	0	0	0	0	8/81
0	1	1	1	0	2/81
0	1	1	0	1	4/81
0	1	0	1	1	4/81
0	1	0	0	0	8/81
0	0	1	1	1	4/81
0	0	1	0	0	8/81
0	0	0	1	0	8/81
0	0	0	0	1	16/81



$$P[D\mid E] = \frac{P[D\cap E]}{P[E]} = \frac{13/81}{41/81} = \frac{13}{41}$$

1.56

Choose a starting point A, so that arc length from A to its neighboring point is largest, especially larger than πr (see graph above). Then each of the rest n-1 points has the probability of 1/2 to be on the same half circle. Since points are put independently, P[rest n-1 points are on the same half circle]=$1/2^{n-1}$.

But any one of the n points could be the starting point, so P[n points are on the same half circle]=$n/2^{n-1}$.