Stat 331 Homework 2 Solutions (30 points possible)

1.59 (4 pts)

(i) $P[(A_1 \cup A_2) \cap (A_3 \cup A_4)] = P[(A_1 \cup A_2)]P[(A_3 \cup A_4)]$

$$= [1 - P(A_1^c \cap A_2^c)]\,[1 - P(A_3^c \cap A_4^c)]$$
$$= [1 - P(A_1^c)P(A_2^c)]\,[1 - P(A_3^c)P(A_4^c)]$$
$$= [1 - (1-p)^2]^2$$

(ii) $P[(A_1 \cap A_2) \cup (A_3 \cap A_4)] = 1 - P[(A_1 \cap A_2)^c \cap (A_3 \cap A_4)^c]$

$$= 1 - P(A_1 \cap A_2)^c\, P(A_3 \cap A_4)^c$$
$$= 1 - [1 - P(A_1 \cap A_2)]\,[1 - P(A_3 \cap A_4)]$$
$$= 1 - [1 - P(A_1)P(A_2)]\,[1 - P(A_3)P(A_4)]$$
$$= 1 - (1 - p^2)^2$$

1.64 (3 pts)

Let: P(S) = P(Survival) = .993
P(C) = P(Cesarean) = .15
$P(C^c)$ = P(Non-Cesarean) = .85
P(S|C) = P(Survival|Cesarean) = .987

We Need to find $P(S|C^c)$

By Corr. 1.6.2:

$P(S) = P(S|C)P(C) + P(S|C^c)P(C^c)$

$.993 = .987(.15) + P(S|C^c)(.85)$ Solving for $P(S|C^c)$ $P(S|C^c) = .994059$

1.65 (2 pts)

Possibilities on a die are {1, 2, 3, 4, 5, 6} each with equal probability of $\frac{1}{6}$
P(Heads) = P(Tails) = $\frac{1}{2}$

$$P(\text{No Heads}) = \tfrac{1}{6}\left[\tfrac{1}{2} + (\tfrac{1}{2})^2 + (\tfrac{1}{2})^3 + (\tfrac{1}{2})^4 + (\tfrac{1}{2})^5 + (\tfrac{1}{2})^6\right] = \tfrac{1}{6}\sum_{i=1}^{6} \tfrac{1}{2^i}$$

1.77 (4 pts)

b) Let: W = You Win
G = He shows a goat
C = Picked a Car

$P(W|G) = P(W|G,C)P(C|G) + P(W|G,C^c)P(C^c|G)$

Note: $P(C|G) = P(C) = \frac{k}{n}$

$P(C^c|G) = 1 - \frac{k}{n}$

$P(W|G,C) = \frac{k-1}{n-2}$

$P(W|G,C^c) = \frac{k}{n-2}$

Therefore: $P(W|G) = \left(\frac{k-1}{n-2}\right)\left(\frac{k}{n}\right)+\left(\frac{k}{n-2}\right)\left(1-\frac{k}{n}\right)=\frac{k^2+n-k}{n^2-2n}$

1.78 (2 pts)

1.92 (6 pts)

a) Mother

Father

	X*	X*
X	XX*	XX*
Y	X*Y	X*Y

P(Daughter is Color-Blind| Mother is C-B and Father is Normal) = 0
P(Son is Color-Blind| Mother is C-B and Father is Normal) = 1

b) Mother

Father

	X*	X
X	XX*	XX
Y	X*Y	XY

P(Daughter is Color-Blind| Both Parents are Normal, Mother's Father was C-B) = 0
P(Son is Color-Blind| Both Parents are Normal, Mother's Father was C-B) = 1/2

1.99 (5 pts)

Explore the possible ways of winning:

1st Trial, can will only with 7 or 11, thus the probability of winning on the first trial is:

$$P(7 \text{ or } 11) = P(7) + P(11) = \frac{8}{36}$$

The Next Trial/s, Can win if you get your point {Your point could be a 4, 5, 6, 8, 9 or 10}

We will employ the Law of Total Probability and the probability that we win before we see a “7” (See page 59 for the derivation)

P(Win)

$$= P(\text{ 7 or 11}) + \frac{P(4)^2}{P(4)+P(7)} + \frac{P(5)^2}{P(5)+P(7)} + \frac{P(6)^2}{P(6)+P(7)} + \frac{P(8)^2}{P(8)+P(7)} + \frac{P(9)^2}{P(9)+P(7)} + \frac{P(10)^2}{P(10)+P(7)}$$

$$= \frac{8}{36} + 2\cdot\frac{1}{36} + 2\cdot\frac{2}{45} + 2\cdot\frac{25}{396} = \frac{244}{495} \approx .49$$

Extra (4 pts)

HH	1 way with probability: $\frac{1}{2^2}$	□$F_1 = 1$
THH	1 way with probability: $\frac{1}{2^3}$	□ $F_2 = 1$
_ T HH	2 ways with probability $\frac{1}{2^4}$	□$F_3 = F_1 + F_2 = 1 + 1 = 2$
_ _ T HH	3 ways with probability $\frac{1}{2^5}$	□ $F_4 = F_3 + F_2 = 2 + 1 = 3$
_ _ _ T HH	5 ways with probability $\frac{1}{2^6}$	□ $F_5 = F_4 + F_3 = 3 + 2 = 5$

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Continuing to do so suggests that the number of flips until we see 2 consecutive heads is:

□ $F_n = F_{n-1} + F_{n-2}$

Therefore we have: $\frac{F_n}{2^{n+1}}$ where n = 2, 3, 4, . . . number of flips

Or we can re-write $\frac{F_n}{2^{n+1}}$ as $\frac{F_{n-1}}{2^n}$