Stat 331 HW5 Solutions

2.68

A= you are the last one left

B= the other customer left after you

$$P[A] = P[A \mid B]P[B] + P[A \mid B^c]P[B^c] = 0 + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Alternative approach:

X= your serving time

Y= the other customer's serving time

Z= the third customer's serving time

$$P[A] = \int_{\substack{x,y,z \in (0,\infty) \\ x > y+z}} f(x,y,z)\,dxdydz = \int_0^\infty \int_0^\infty \int_{y+z}^\infty \lambda^3 e^{-\lambda(x+y+z)}\,dxdydz = \frac{1}{4}$$

Additional

Density of Gamma (α, β)

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}, x \geq 0$$

$$M_X(t) = E[e^{tx}] = \int_0^\infty e^{tx} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} dx = \frac{1}{\Gamma(\alpha)} \int_0^\infty \frac{1}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\frac{x}{\beta}(1-\beta t)} dx$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^\infty \left(\frac{1}{1-\beta t}\right)^\alpha \left[\frac{x}{\beta}(1-\beta t)\right]^{\alpha-1} e^{-\frac{x}{\beta}(1-\beta t)} d\left[\frac{x}{\beta}(1-\beta t)\right]$$

$$= \left(\frac{1}{1-\beta t}\right)^\alpha, t < \frac{1}{\beta}$$

$$E[x] = M'_X(0) = \alpha\beta$$

$$\mathrm{var}[x] = M''_X(0) - [M'_X(0)]^2 = \alpha\beta^2$$