STAT 581 Homework No.1

1. If a multivariate function has continuous partial derivatives, the order in which the derivatives are calculated dose not matter. Thus, for example, the function f(x, y) of two variables has equal third partials
$$\frac{\partial^3}{\partial x^2 \partial y} f(x,y) = \frac{\partial^3}{\partial y \partial x^2} f(x,y).$$
   (a) How many fourth partial derivatives does a function of three variables have?
   (b) Prove that a function of n variables has $\binom{n+r-1}{r}$ r th partial derivatives.

2. My telephone rings 12 times each week, the calls being randomly distributed among the 7 days. What is the probability that I get at least one call each day?
3. A closet contains n pairs of shoes. If $2r$ shoes are chosen at random ($2r < n$), what is the probability that there will be no matching pair in the sample?
4. The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?
5. A fair die is cast until a 6 appears. What is the probability that is must be cast more than five times?
6. IF the probability of hitting a target is 0.2, and ten shots are fired independently, what is the probability of the target being hit at least twice? What is the conditional probability that the target is hit at least twice, given that it is hit at least once?
7. Prove that the following functions are cdfs.
   (a). $(1+e^{-x})^{-1}, x \in (-\infty, \infty)$
   (b). $e^{-e^{-x}}, x \in (-\infty, \infty)$
8. (a) Let X be a continuous, nonnegative random variable $[f(x) = 0 \, for x < 0]$. Show that
$$Ex = \int_0^\infty [1 - F_X(x)]dx,$$
   Where $F_X(x)$ is the cdf of X.
   (b) Let X be a discrete random variable whose range is the nonnegative integers. Show that
$$Ex = \sum_{k=0}^{\infty} (1 - F_X(k)),$$
   Where $F_X(k) = P(X \le k)$. Compare this with part (a).
9. Betteley (1977) provides an interesting addition law for expectations. Let X and Y be any two random variables and define
$$X \wedge Y = \min(X,Y) \quad \text{and} \quad X \vee Y = \max(X,Y).$$
   Analogous to the probability law $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, show that
$$E(X \vee Y) = EX + EY - E(X \wedge Y)$$
   (Hint: Establish that $X + Y = (X \vee Y) + (X \wedge Y)$.)

10. Compute E X and Var X for each of the following probability distributions.

    (a). $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$

11. The random pair (X, Y) has the distribution

		X		
		1	2	3
Y	2	1/12	1/6	1/12
	3	1/6	0	1/6
	4	0	1/3	0

    (a) Show that X and Y are dependent.
    (b) Give a probability table for random variables U and V that have the same marginals as X and Y but are independent.

12. Let X and Y but independent standard normal random variables. Show that X/(X+Y) has a Cauchy distribution.

13. Let X ~ Poisson (θ), Y ~ Poisson(λ), independent. It was shown in Theorem 4.3.2 that the distribution of X + Y is Poisson$(\theta + \lambda)$. Show that the distribution of X|X+Y is binomial with success probability $\theta/(\theta + \lambda)$. What is the distribution of Y|X+Y?