## Stat581 Midterm Solutions

### 1.

Since $A = \sum_{i=1}^{\infty} A_i$, $P(A) = \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} p(1-p)^{i-1} = 1$.

### 2.

$$F_X(x) = P(X \leq x) < P(Y \leq x) = F_Y(x).$$

### 3.

Suppose $A \in \mathcal{B}$ for some prob. space $(\Omega, \mathcal{B}, P)$,

$$\begin{aligned} 1_A^{-1}(\emptyset) &= \emptyset \in \mathcal{B}, \\ 1_A^{-1}(\{1\}) &= A \in \mathcal{B}, \\ 1_A^{-1}(\{0\}) &= A_c \in \mathcal{B}, \\ 1_A^{-1}(\{0,1\}) &= \Omega \in \mathcal{B}. \end{aligned}$$

### 4.

$$Z(\omega) = X(\omega) 1_A(\omega).$$

### 5.

For $A_i, i = 1, 2, ...,$ consider $B_i = A_i \backslash \cup_{j=1}^{i-1} A_j$.

**6.**

$\sigma(\mathcal{C})$ has $2^4$ elements.

**7.**

$$\begin{aligned} P(\Omega \backslash \cup_{n\geq 1} A_n) = 1 \quad &\Rightarrow \quad P((\cup_{n\geq 1} A_n)^c) = 1 \\ &\Rightarrow \quad P(\cup_{n\geq 1} A_n) = 0 \\ &\Rightarrow \quad P(A_n) = 0, \forall n. \end{aligned}$$

**8.**

$$A_{n+1} \subset A_n \Rightarrow \cup_{k\geq n} A_k = A_n \Rightarrow \limsup A_n = \cap_{n\geq 1} A_n,$$

$$\cap_{k\geq 1} A_k \subset \cap_{k\geq n} A_k, \forall n \Rightarrow \cap_{k\geq 1} A_k \subset \cup_{n\geq 1} \cap_{k\geq n} A_k = \liminf A_n \leq \limsup A_n = \cap_{k\geq 1} A_k$$

**9.**

Since $\mathbb{Q} \cap [0,1] \in \mathcal{B}([0,1])$, by problem 3, $1_{(\omega\in\mathbb{Q}\cap[0,1])} \in \mathcal{B}([0,1])$.