A SCIENTIFIC WRITING EXAMPLE

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Abstract

We investigate some extinction problems in Markov branching processes.

Keywords: Branching process; Markov process

1 Introduction

2 Markov branching processes

A MBP Z_t is completely determined by its offspring distribution and expected life span. Suppose the life length of each particle follows exponential distribution with parameter a, and the probability generating function (pgf) of the offspring distribution is given by $f(s) = \sum_{k=0}^{\infty} p_k s^k$. Denote the pgf of the process Z_t by $F(s,t) = E_1\left[s^{Z_t}\right]$, then $F(s,t)$ satisfies the forward Kolmogorov equation

$$\frac{\partial F(s,t)}{\partial t} = \phi(s)\frac{\partial F(s,t)}{\partial s},$$

where $\phi(s) = a(f(s) - s)$, and the backward Kolmogorov equation

$$\frac{\partial F(s,t)}{\partial t} = \phi(F(s,t)). \tag{1}$$

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Theorem. *Suppose Z_t is a Markov branching process, define the survival probability $Q(t) = P_1(Z_t > 0)$. In the subcritical case,*

$$Q(t) \sim b^{-1}e^{-rt}, \text{ as } t \to \infty. \tag{2}$$

Proof. By Equation (1).
This result is confirmed by Figure 1.

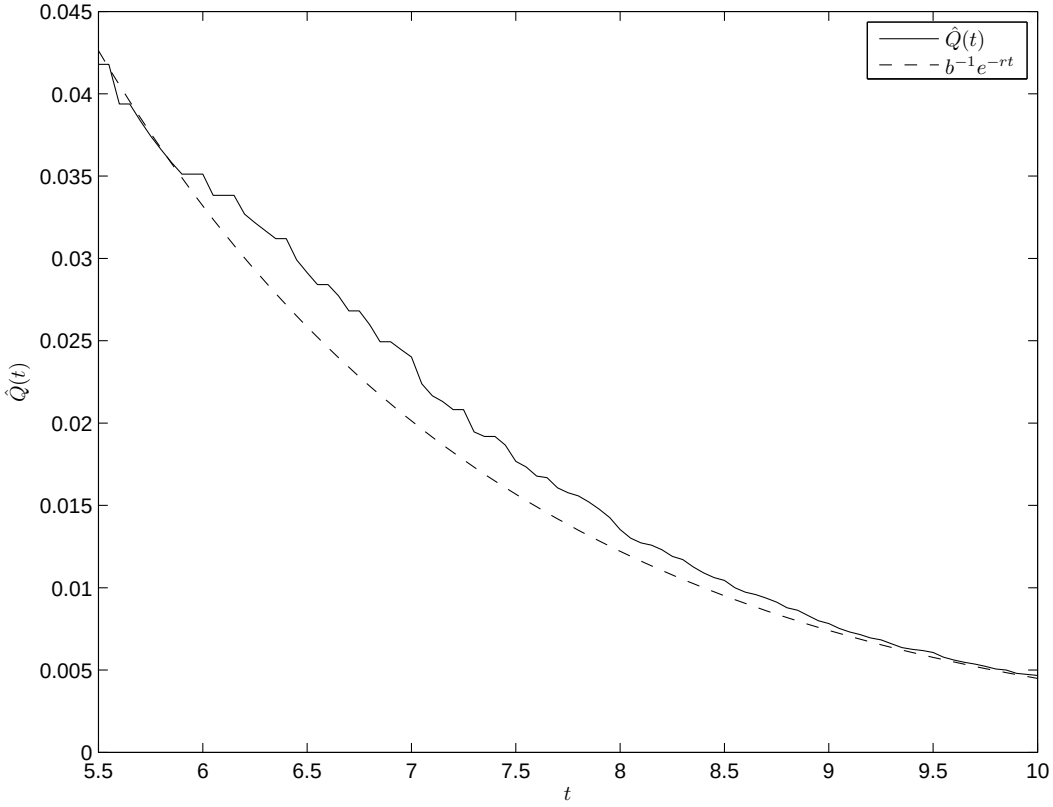

Figure 1: Survival frequency $\hat{Q}(t)$ in a MBP, based on 500 simulations and its asymptotic equivalent function.

3 Acknowledgements