Restricted Estimation of the Cumulative Incidence Functions Corresponding to \( k \) Competing Risks

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Abstract

In the competing risks problem an important role is played by the cumulative incidence function (CIF), whose value at time \( t \) is the probability of failure by time \( t \) for a particular type of failure in the presence of other risks. Its estimation and asymptotic distribution theory have been studied by many. In some cases there are reasons to believe that the CIF’s due to \( k \) types of failure are order restricted. Several procedures have appeared in the literature for testing for such orders. In this paper we initiate the study of estimation of \( k \) CIF’s subject to a type of stochastic ordering, both when there are just \( k \) causes of failure and when there are more than \( k \) causes of failure, treating those other than the \( k \) of interest as a censoring mechanism. Weak convergence results for the estimators have been derived. It is shown that when the order restriction is strict, the asymptotic distributions are the same as those for the empirical estimators without the order restriction. Thus we get the restricted estimators “free of charge”, at least in the asymptotic sense. A test to test for equality of the \( k \) CIF’s against the alternative that are ordered will also be given and an example will be discussed to illustrate the theory.