1. Let $X_1, X_2, \ldots$ be i.i.d. random variables having an exponential distribution with mean 1. Show that

a. $P(X_n > \log n \text{ i.o.}) = 1$

b. $P(X_n > n \text{ i.o.}) = 0$.

2. The Hewitt-Savage 0-1 Law does not hold if the $X_k$ are independent but not identically distributed. Give an example of independent $X_1, X_2, \ldots$ and a symmetric event in $\sigma(X_1, X_2, \ldots)$ which has probability strictly between 0 and 1. *Hint:* Try to find an event involving the value of the sum $\sum_k X_k$ such that this event is determined by the value of $X_1$ alone.

3. Let $(\Omega, \mathcal{B}, P) = ([0,1], \text{Borel sets, Lebesgue measure})$. Define a random variable $X$ on this space such that $X$ has a uniform distribution on $[-1,1]$, i.e. such that $P(a \leq X \leq b) = (b - a)/2$ for $-1 \leq a \leq b \leq 1$. Use the definition of expectation to show that $E[X] = 0$.

4. Use Borel-Cantelli to construct examples of sequences of random variables $X_1, X_2, \ldots$ such that

a. $X_n \rightarrow 0 \text{ a.s. and } E[X_n] \equiv 1$.

b. $X_n \rightarrow 0 \text{ a.s., } E[X_n] < 1 \text{ for all } n \text{ and } E[X_n] \rightarrow 1$.

c. $X_n \rightarrow \infty \text{ a.s. and } E[X_n] \rightarrow -\infty$.

d. $X_n \not\rightarrow 0 \text{ a.s. and } E[X_n] \rightarrow 0$.