Assignment 2, STAT 582, due February 18

1a. Let $X_1, X_2, \ldots$ be independent with the same finite mean $\mu$ and the same variance $\sigma^2 < \infty$ (but not necessarily i.i.d.). Show that $S_n/n \xrightarrow{P} \mu$.

b. Now drop the variance assumption: Let $X_1, X_2, \ldots$ be independent with the same finite mean $\mu$. Give an example which shows that $S_n/n \xrightarrow{P} \mu$ may not hold even if $Var[X_j] < \infty$ for all $j$. Hint: Let $X_j$ be symmetric around 0 in a way such that condition (ii) in Theorem 7.2.1 is violated.

2. Let $X_1, X_2, \ldots$ be independent with $X_j$ having a uniform distribution on $[0,j]$. Show that $S_n/n^2$ converges in probability and determine the limit.

3. Let $X_1, X_2, \ldots$ be independent such that $X_j$ has density $f_j(x) = c_j/x^3$ for $x \geq 1/j$.

   a. Determine the constants $c_j$.

   b. Let $S_n = \sum_{j=1}^{n} X_j$. Show that $S_n/\log n$ converges in probability and determine the limit.