2.5

2a
\[ P(G) = P(A \cap G) + P(B \cap G) \]
\[ = P(A)P(G|A) + P(B)P(G|B) \]
\[ = .4 \times .85 + .6 \times .75 = .79 \]

b
\[ P(A|G) = \frac{P(A \cap G)}{P(G)} = \frac{.4 \times .85}{.79} = .43 \]

You could also make a diagram like this and use 1000 as the total number of seeds

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germ</td>
<td>340</td>
<td>450</td>
<td>790</td>
</tr>
<tr>
<td>not</td>
<td>60</td>
<td>150</td>
<td>210</td>
</tr>
<tr>
<td>total</td>
<td>400</td>
<td>600</td>
<td>1000</td>
</tr>
</tbody>
</table>

We are given that one chip in the new sample is blue. Therefore this new sample can be thought of choosing 4 chips from a sample of 9 where 2 chips are from each color.

we get \( \frac{\binom{2}{2} \binom{4}{2}}{\binom{9}{4}} = \frac{5}{14} = .35714 \)

3.1

3

a) \(10 \times (1+2+3+4)c = 1\)

b) \(1/55\) use formula from e

c) sum of geometric series \( \frac{1}{1-\frac{1}{4}}c = 1 \)

d) \(1/30 \times (1+4+9+16)c = 1\)

e) \(\frac{1}{(2+13)}\) answer in back of book is wrong

7

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>11/36</td>
<td>9/36</td>
<td>7/36</td>
<td>5/36</td>
<td>3/36</td>
<td>1/36</td>
</tr>
</tbody>
</table>

1
3.2

4) \[ E(X) = -1 \times \frac{4}{9} + 0 \times \frac{1}{9} + 1 \times \frac{4}{9} = 0 \]
\[ E(X^2) = -1^2 \times \frac{4}{9} + 0^2 \times \frac{1}{9} + 1^2 \times \frac{4}{9} = \frac{8}{9} \]
\[ E(3X^2 - 2X + 4) = 9 \times \frac{4}{9} + 4 \times \frac{1}{9} + 5 \times \frac{4}{9} = \frac{60}{9} \]

or we could compute the last one by doing \[ 3E(X^2) - 2E(X) + 4 = 3 \times \frac{8}{9} - 2 \times 0 + 4 = \frac{60}{9} \]

8) \[ E(X) = \sum x \cdot \frac{n}{x^2} = \sum \frac{6}{x^2} = \frac{6}{2} \sum \frac{1}{x} \text{ but does not converge by the p-series test and this sum is equal to infinity.} \]

19) a) \[ E(X) = (1 \times 2^5) + 2(2^4) + 3(2^3) + 4(2^2) + 5(2) + 6 + 7)/64 \]

mean = \( \frac{127}{64} \)
\[
E(X^2) = (1^2(2^2)+2^2(2^4)+3^2(2^2)+4^2(2^2)+5^2(2)+6^2+7^2)/64 = 367/64 \text{var}(X) = \]
\[
E(X^2) - E(X)^2 = 367/64 - (127/64)^2 = 1.7966
\]
\[
sd(X) = \sqrt{\text{var}(X)} = 1.34
\]
\[
\text{Skewness, we know than } \mu = 127/64 \text{ so } E((X-\mu)^3) = ((1-\mu)^3 * 2^5 + (2-\mu)^3 * 2^4 + ... + (7-\mu)^3)/64 = 4.0061
\]
\[
\text{skew} = \frac{4.0061}{1.34} = 1.664
\]
b) Use the exact same techniques as you did in part a
\[
E(X) = 6.016
\]
\[
\text{Var}(X) = 1.34
\]
\[
\text{Skew}(X) = -1.664
\]
20) a)
\[
f(x) = p(X = x) = \frac{6x+33(x-6-x)}{49} \text{, } x = 1, 2, 3, 4, 5, 6
\]
\[
|\text{x}||\text{P(X=x)}|
\[
|0||.436|
|1||.413|
|2||.132|
|3||.0177|
|4||.00096|
|5||.000018|
|6||.000000072|
\]
b) \[
E(X) = \sum xp(X = x) = 0(.436)+1(.413)+2(.132)+3(.0177)+4(.00096)+5(0.000018) + 6(.000000072) = .7347
\]
\[
\text{Var}(X) = \sum (x-\mu)^2p(X = x) = .5776
\]
\[
\text{sd}(X) = \sqrt{\text{Var}(X)} = \sqrt{.5776} = .76
\]
c) Look at table P(X=0) = .436 is the max so the most likely value for X is 0.
d) 3 out of 2500000 got 6 numbers correct we would expect 25000000*P(X=6)=1.788 winners
for 5 numbers we expect 25000000*P(X=5)=461.25 and 390 won
4 numbers we expect 25000000*P(X=4)=24215.5 and 22187
Although these numbers are not exact they are considered reasonable.
e) use same technique as in d, just now for all values of X
\[
|\text{x}||\text{expected}||\text{actual}|
\[
|0||60.16||65|
|1||57||55|
|2||18.27||16|
|3||2.44||2|
\]
Again the results are reasonable
25
Each game has a negative expected value per one dollar bet. Therefore, each game on average is going to lose a certain amount per time played. Thus, the E(N) is equal to 5/E(X). Where E(X) is the expected loss per time played a game on a one dollar bet.
a) For the state lottery \[
E(X) = -1(999/1000) + 499(1/1000) = -.5
\]
Using the formula from above we get \[E(N) = \frac{5}{-.5} = 10\]
b) Chuck-a-luck \[E(X) = (1(75) + 2(15) + 3(1) - 1(125))/216 = -.19/216\]
\[ E(N) = \frac{5}{17/216} = 63.53 \]
c) Roulette American 
\[ E(X) = 1(18/38) - 1(20/38) = -2/38 \]
\[ E(N) = 5/(2/38) = 95 \]
France 
\[ E(X) = 1(18/37) - 1(19/37) = -1/37 \]
\[ E(N) = 5/(1/37) = 185 \]
d) Craps 
\[ E(X) = 1(18/36) - 1(24/36) = -2/36 \]
\[ E(N) = 5/(-.01414) = 353.71 \]
e) High low if you choose low there are 15 outcomes of the sum of 2 dice that add up to numbers between 2 and 6. 
\[ E(X) = 1(15/36) - 1(21/36) = -1/6 \]
\[ E(N) = 5/(-1/6) = 30 \]

3.3
4) a) Probability that X is at most 5 equals 
\[ P(X < 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \]
\[ P(X = x) = 12cx \times .45^{x} \times .55^{12-x} \]
Using this formula we get 
\[ P(X < 6) = .5269 \]
b) \[ P(X > 5) = 1 - P(X < 5) = 1 - .5269 = .4731 \]
c) \[ P(X = 7) \] just use formula from a and get \[ .149 \]
d) Using formulas from book 
\[ E(X) = np = 12 \times .45 = 5.4 \]
\[ Var(X) = np(1-p) = 5.4 \times .55 = 2.97 \]
\[ SD(X) = \sqrt{Var(X)} = \sqrt{2.97} = 1.723 \]

7
a) Each trial is a success (in A) or failure (not in A) and each trial is independent of each other. This is a binomial distribution with parameters n=2000 and p= area of A divided by total area. Area of A= quarter area of circle with radius 1. A = \[ \frac{\pi}{4} \]. Total area = 1*1. So W is distributed as bin(2000, \[ \frac{\pi}{4} \])
b) using same formulas from problem 4.
\[ E(X) = 1570.96 \]
\[ Var(X) = 337.096 \]
\[ SD(X) = 18.36 \]
c) From properties of expectation \[ E(aX) = aE(X) \] where a is any constant. So
\[ E(W/500) = E(W)/500 = 1570.96/500 = \pi \]
d) The code from matlab is as follows

```matlab
function value = pairpi;
int count;
count = 0;
for (k = 1:2000)
x = rand;
y = rand;
```

4
if (x^2+y^2<1)
count = count + 1;
end
end
count/2000*4

The end result should give you a number close to pi.
e) We could use the equation $x^2 + y^2 + z^2 = 1$ and find all the points inside this volume. On the computer we would just pick uniform random numbers from zero to one for $x$, $y$, and $z$ and see if the square of each of these added together is less than 1. The volume of a sphere is $\frac{4}{3}\pi r^3$ and would fit in a box of dimension $2*2*2=8$ cubic units. So the proportion of the sphere in the cube is . So we could estimate the volume of a sphere by finding an estimate of $\pi$ using similar reasoning from part d) just adding a $z$ squared term to our equation.
f) The n-dimensional sphere is the set of all points $(x1,x2,x3,...xn)$ such that $x1^2 + x2^2 + ... + xn^2 = 1$ We could now generate n random numbers from zero to one, for the n x’s and count all the points that would make the above equation less than one. The volume equation for an n-dimensional cube is $V_n = \pi^{n/2}/\Gamma(n/2 + 1)$