On the Chernoff bound for efficiency of quantum hypothesis testing

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The quantum statistic is a rapidly growing area of modern statistics (see [1] and [2]). Why? Today, quantum states can be manufactured. For example, in one method ions are placed in a trap created by electrostatic potential and radio-frequency oscillations. The ions then are cooled by laser emission, and arranged on a line in the trap. After that, each individual ion can be accessed by laser pulses and their joint quantum state can be altered according to the researcher's wishes. This ability to built and manipulate quantum systems is changing our thinking about computation and information transmission. Suddenly, certain classic problems – the factorization of large integers, the search in an unstructured database, secure communication – are not as difficult as they always were.

This conceptual change also affects statistics.

For example, how can a quantum state manufacturer check if states have been generated faithfully? We can anticipate the statistician's answer: Select a sample of the states and perform a statistical test. But now, besides designing the test, the statistician must play an additional role, the role of advisor on how to perform measurements of a sample of quantum states. Since in quantum mechanics both the measurement and the state determine the probability distribution of outcomes, the choice of measurement affects the properties of the statistical test. The paper contributes by estimating the Chernoff-Hoeffding rate for the efficiency of testing hypotheses about quantum states.

On of the open problem of quantum statistics is to find out whether the joint measurements on a sample of quantum states can outperform separate measurements of each state in the sample. This paper derives exact expressions and bounds on the Chernoff-Hoeffding rates of testing efficiency for both joint and separate measurements. The exact espressions are derived if at least one of the states is pure and the approximate bounds are given if both states are mixed. The rates and bounds are given in terms of the geometrical properties of quantum states. The derived expressions are then used to compare efficiency of joint and separate measurements.

In detail, for the case of joint measurements and pure states, the asymptotic rate is,

$$\frac{1}{N}\log R \sim 2\log\left|\langle\psi_0|\ \psi_1\rangle\right| \text{ as } N\to\infty. \tag{1}$$

If only one of the states is pure then

$$\frac{1}{N}\log R \sim \log\langle\psi_0|\,\rho_1\,|\psi_0\rangle\,, asN\to\infty. \tag{2}$$

If both states are mixed then the following inequality holds

$$2\log F\left(\rho_0,\rho_1\right) \lesssim \frac{1}{N}\log R \lesssim \log F\left(\rho_0,\rho_1\right),\ \text{ as } N\to\infty, \tag{3}$$

where

$$F(\rho_0,\rho_1) = \mathrm{tr}\sqrt{\sqrt{\rho_0}\rho_1\sqrt{\rho_0}} \tag{4}$$

is fidelity between two states.

For separate measurements there is a measurement with a simple structure and the optimal asymptitotic rate. In caseof pure quantum states the asymptotic rate is the same as for joint measurements. For the mixed states it is proved that

$$\frac{1}{N}\log R \lesssim \log F(\rho_0,\rho_1). \tag{5}$$

The results are illustrated by a test of quantum entanglement.

References

[1] O. E. Barndorff-Nielsen, R. D. Gill and P. E. Jupp. (2003) *On Quantum Statistical Inference*, Journal of the Royal Statistical Society: Series B (Statistical Methodology), v. 65(4), 775-805

[2] R. D. Gill. (2001) *Asymptotics in Quantum Statistics*, in State of the Art in Probability and Statistics, Volume 36 in IMS Lecture Notes - Monograph Series, 255-285