Tests About Proportions

Suppose $X_1, \ldots, X_n \sim \text{binom}(1, p)$

We would like to test

$$H_0 : p = p_0$$

versus an alternative such as

$$H_1 : p \neq p_0 \,, \quad H_1 : p > p_0 \,, \quad H_1 : p < p_0 \,.$$

Let $Y = \sum_{i=1}^{n} X_i$ (# of successes)

Recall $E[X_i] = p$

$\text{Var}(X_i) = p(1-p)$.

By the CLT,

$$\frac{Y/n - p_0}{\sqrt{p_0(1-p_0)/n}} \approx N(0,1).$$

Therefore, the test that accepts H_0 when

$$\frac{|Y/n - p_0|}{\sqrt{p_0(1-p_0)/n}} < z_{\alpha/2}$$

is a level of significance α test

H_0 vs. $H_1 : p \neq p_0$.