

Stat310 HW2 Solutions (100')

5'

$$\begin{aligned} \text{1.5-2 (a)} \quad P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

$$\text{(b)} \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.6} = 0.$$

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$$\text{1.5-12 (a)} \quad \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2;$$

$$\text{(b)} \quad \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2;$$

$$\text{(c)} \quad \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2;$$

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$$\text{(d)} \quad \frac{5!}{3!2!} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2.$$

$$\text{1.5-14 (a)} \quad 1 - (0.4)^3 = 1 - 0.064 = 0.936;$$

$$\text{(b)} \quad 1 - (0.4)^8 = 1 - 0.00065536 = 0.99934464.$$

$$\text{1.5-16 (a)} \quad \sum_{k=0}^{\infty} \frac{1}{5} \left(\frac{4}{5}\right)^{2k} = \frac{5}{9};$$

$$\text{(b)} \quad \frac{1}{5} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{3}{5}.$$

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$$\begin{aligned} \text{1.6-2 (a)} \quad P(G) &= P(A \cap G) + P(B \cap G) \\ &= P(A)P(G|A) + P(B)P(G|B) \\ &= (0.40)(0.85) + (0.60)(0.75) = 0.79; \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad P(A|G) &= \frac{P(A \cap G)}{P(G)} \\ &= \frac{(0.40)(0.85)}{0.79} = 0.43. \end{aligned}$$

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1.6-8 Let A be the event that the VCR is under warranty.

$$\begin{aligned}
 P(B_1 | A) &= \frac{(0.40)(0.10)}{(0.40)(0.10) + (0.30)(0.05) + (0.20)(0.03) + (0.10)(0.02)} \\
 &= \frac{40}{40 + 15 + 6 + 2} = \frac{40}{63} = 0.635; \\
 P(B_2 | A) &= \frac{15}{63} = 0.238; \\
 P(B_3 | A) &= \frac{6}{63} = 0.095; \\
 P(B_4 | A) &= \frac{2}{63} = 0.032.
 \end{aligned}$$

1.6-10 (a) $P(AD) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674;$

(b) $P(N | AD) = \frac{0.0490}{0.0674} = 0.727; P(A | AD) = \frac{0.0184}{0.0674} = 0.273;$

(c) $P(N | ND) = \frac{(0.98)(0.95)}{(0.02)(0.08) + (0.98)(0.95)} = \frac{9310}{16 + 9310} = 0.998;$

$P(A | ND) = 0.002.$

(d) Yes, particularly those in part (b).

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2.1-2 (a)

$$f(x) = \begin{cases} 0.6, & x = 1, \\ 0.3, & x = 5, \\ 0.1, & x = 10. \end{cases}$$

(b)

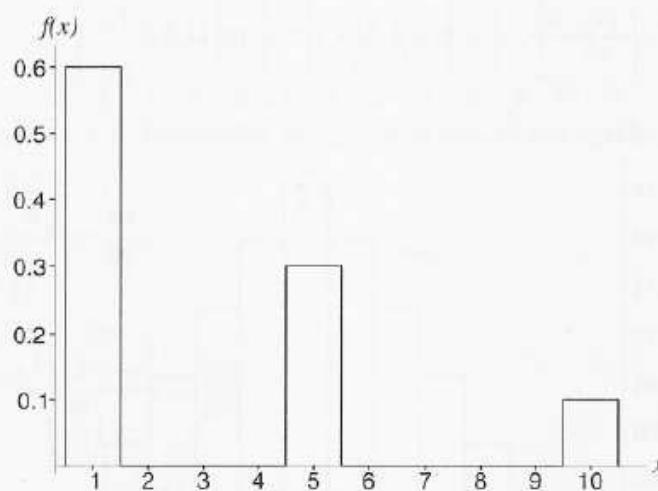


Figure 2.1-2: A Probability Histogram

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2.1-8 (a) The space of W is $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

$$P(W = 0) = P(X = 0, Y = 0) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}, \text{ assuming independence.}$$

$$P(W = 1) = P(X = 0, Y = 1) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}.$$

Continuing this, we see that $f(w) = P(W = w) = \frac{1}{12}, w \in S$.

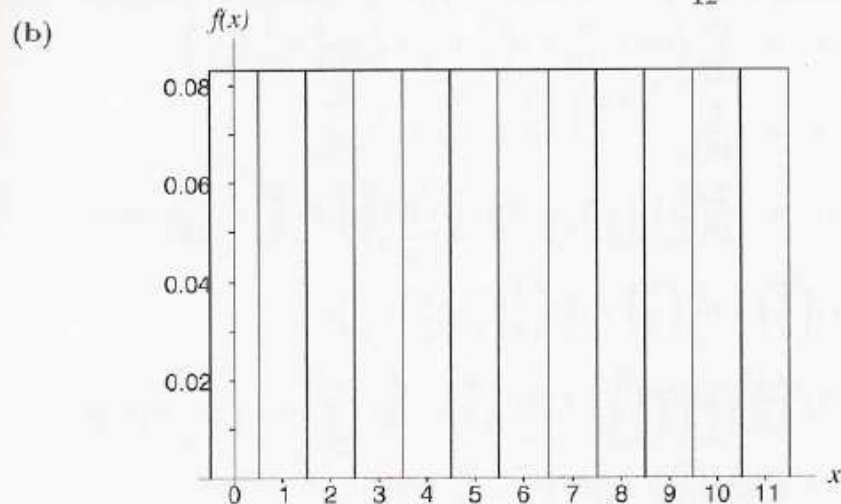


Figure 2.1-8: Probability histogram of sum of two special dice

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$$\mathbf{2.1-12} \quad OC(0.04) = \frac{\binom{1}{0} \binom{24}{5}}{\binom{25}{5}} + \frac{\binom{1}{1} \binom{24}{4}}{\binom{25}{5}} = 1.000;$$

$$OC(0.08) = \frac{\binom{2}{0} \binom{23}{5}}{\binom{25}{5}} + \frac{\binom{2}{1} \binom{23}{4}}{\binom{25}{5}} = 0.967;$$

$$OC(0.12) = \frac{\binom{3}{0} \binom{22}{5}}{\binom{25}{5}} + \frac{\binom{3}{1} \binom{22}{4}}{\binom{25}{5}} = 0.909;$$

$$OC(0.16) = \frac{\binom{4}{0} \binom{21}{5}}{\binom{25}{5}} + \frac{\binom{4}{1} \binom{21}{4}}{\binom{25}{5}} = 0.834.$$

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$$\begin{aligned} \mathbf{2.2-2} \quad 1 &= \sum_{x=0}^6 f(x) = \frac{9}{10} + c \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right) \\ c &= \frac{2}{49}; \\ E(\text{Payment}) &= \frac{2}{49} \left(1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{1}{6} \right) = \frac{71}{490} \text{ units.} \end{aligned}$$

$$\mathbf{2.2-4} \quad E(X) = (-1) \left(\frac{4}{9} \right) + (0) \left(\frac{1}{9} \right) + (1) \left(\frac{4}{9} \right) = 0;$$

$$E(X^2) = (-1)^2 \left(\frac{4}{9} \right) + (0)^2 \left(\frac{1}{9} \right) + (1)^2 \left(\frac{4}{9} \right) = \frac{8}{9};$$

$$E(3X^2 - 2X + 4) = 3 \left(\frac{8}{9} \right) - 2(0) + 4 = \frac{20}{3}.$$

$$\mathbf{2.2-6} \quad E(X) = \$499(0.001) - \$1(0.999) = -\$0.50.$$

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$$\mathbf{2.2-12} \quad (1) \cdot \frac{15}{36} + (-1) \cdot \frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(1) \cdot \frac{15}{36} + (-1) \cdot \frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(4) \cdot \frac{6}{36} + (-1) \cdot \frac{30}{36} = \frac{-6}{36} = \frac{-1}{6}.$$