**2.3-4** $E[(X-\mu)/\sigma] = (1/\sigma)[E(X)-\mu] = (1/\sigma)(\mu-\mu) = 0;$

$$E\{[(X-\mu)/\sigma]^2\} = (1/\sigma^2)E[(X-\mu)^2] = (1/\sigma^2)(\sigma^2) = 1.$$

**2.3-6** **(a)** $f(x) = P(X=x) = \dfrac{\binom{6}{x}\binom{43}{6-x}}{\binom{49}{6}}, \qquad x = 0,1,2,3,4,5,6;$

**(b)**
$$\begin{aligned}\mu_X &= \sum_{x=0}^{6} x f(x) = \frac{36}{49} = 0.7347, \quad n\left(\frac{N_1}{N}\right)\\ \sigma_X^2 &= \sum_{x=0}^{6}(x-\mu)^2 f(x) = \frac{5{,}547}{9{,}604} = 0.5776; \quad n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)\\ \sigma_X &= \frac{43}{98}\sqrt{3} = 0.7600;\end{aligned}$$

**(c)** $f(0) = \dfrac{435{,}461}{998{,}844} > \dfrac{412{,}542}{998{,}844} = f(1); \quad X = 0$ is most likely to occur.

**(d)** The numbers are reasonable because

$$\begin{aligned}(25{,}000{,}000)f(6) &= 1.79;\\ (25{,}000{,}000)f(5) &= 461.25;\\ (25{,}000{,}000)f(4) &= 24{,}215.49;\end{aligned}$$

**(e)** The respective expected values, $(138)f(x)$, for $x = 0,1,2,3$, are 60.16, 57.00, 18.27, and 2.44, so the results are reasonable. See Figure 2.3-6 for a comparison of the theoretical probability histogram and the histogram of the data.



2.3-10 a) $P(X \geq 1) = \dfrac{2^1}{\binom{3}{1}} = \dfrac{2}{3}$



**(b)** $\displaystyle\sum_{k=1}^{5} P(X \geq k) = P(X=1) + 2P(X=2) + \cdots + 5P(X=5) = \mu;$

**(c)** $\mu = \dfrac{5,168}{3,465} = 1.49149;$

**(d)** In the limit, $\mu = \dfrac{\pi}{2}$.

**2.3-12** $\overline{x} = \dfrac{409}{50} = 8.18.$

**2.3-14** $f(1) = \dfrac{3}{8},\ f(2) = \dfrac{2}{8},\ f(3) = \dfrac{3}{8}$

$$\mu = 1\cdot\frac{3}{8} + 2\cdot\frac{2}{8} + 3\cdot\frac{3}{8} = 2,$$

$$\sigma^2 = 1^2\cdot\frac{3}{8} + 2^2\cdot\frac{2}{8} + 3^2\cdot\frac{3}{8} - 2^2 = \frac{3}{4}.$$

**2.4-4** **(a)** $P(X \le 5) = 0.5269$;

**(b)** $P(X \ge 6) = 1 - P(X \le 5) = 0.4731$;

**(c)** $P(X \le 7) - P(X \le 6) = 0.8883 - 0.7393 = 0.1490$;

**(d)** $\mu = (12)(0.45) = 5.4,\ \sigma^2 = (12)(0.45)(0.55) = 2.97,\ \ \sigma = \sqrt{2.97} = 1.723$.

**2.4-6** **(a)** $X$ is $b(7,\ 0.15)$;

**(b)**
$$\begin{aligned}
\text{(i)}\quad P(X \ge 2) &= 1 - P(X \le 1) = 1 - 0.7166 = 0.2834;\\
\text{(ii)}\quad P(X = 1) &= P(X \le 1) - P(X \le 0) = 0.7166 - 0.3206 = 0.3960;\\
\text{(iii)}\quad P(X \le 3) &= 0.9879.
\end{aligned}$$

**2.4-8** **(a)** $X$ is $b(15, 0.2)$,

**(b)** $\mu = 15(0.2) = 3,\ \ \sigma^2 = 15(0.2)(0.8) = 2.4,\ \ \sigma = \sqrt{2.4} = 1.549$;

**(c)** $P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.8358 = 0.1642$.

**2.4-10** **(a)** $X$ is $b(6,\ 0.05)$;

**(b)** $\mu = 6(0.05) = 0.3;\ \ \sigma^2 = 6(0.05)(0.95) = 0.285$;

**(c)**
$$\begin{aligned}
\text{(i)}\quad P(X = 0) &= 0.7351;\\
\text{(ii)}\quad P(X \le 1) &= 0.9672;\\
\text{(iii)}\quad P(X \ge 2) &= 1 - P(X \le 1) = 0.0328.
\end{aligned}$$

**2.4-12** **(a)** $\mu = 14(0.55) = 7.7,\ \sigma^2 = 14(0.55)(0.45) = 3.465$;

so $n = 29$.

**2.4-20** $$\frac{(0.1)(1 - 0.95^5)}{(0.4)(1 - 0.97^5) + (0.5)(1 - 0.98^5) + (0.1)(1 - 0.95^5)} = 0.178.$$

**2.4-22** **(a)** $1 - 0.01^4 = 0.99999999$; **(b)** $0.99^4 = 0.960596$.

2.5-3

$\mu = M'(0)$

$M(t) = 2/5\, e^{t} + \frac{1}{5} e^{2t} + 2/5\, e^{3t}$

$M'(t) = 2/5\, e^{t} + 2/5\, e^{2t} + 6/5\, e^{3t}$

$M''(t) = 2/5\, e^{t} + 4/5\, e^{2t} + 18/5\, e^{3t}$

$M'(0) = 2/5 + 2/5 + 6/5 = 10/5 = \boxed{2 = \mu}$

$M''(0) = 2/5 + 4/5 + 18/5 = 24/5 = E(x^2)$

$\sigma^2 = E(x^2) - [E(x)]^2 = 24/5 - 4 = \boxed{4/5 = \sigma^2}$

$$f(x) = \begin{cases} 2/5 & x = 1 \\ 1/5 & x = 2 \\ 2/5 & x = 3 \end{cases}$$

**2.5-8** $\binom{10-1}{5-1}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{126}{1024} = \frac{63}{512}$.

**2.5-10** **(a)** Negative binomial with $r = 10, p = 0.6$ so

$$\mu = \frac{10}{0.60} = 16.667,\ \sigma^2 = \frac{10(0.40)}{(0.60)^2} = 11.111,\ \sigma = 3.333;$$

**(b)** $P(X = 16) = \binom{15}{9}(0.60)^{10}(0.40)^6 = 0.1240.$

**2.5-12**

$$\begin{aligned} P(X > k + j \,|\, X > k) &= \frac{P(X > k + j)}{P(X > k)} \\ &= \frac{q^{k+j}}{q^k} = q^j = P(X > j). \end{aligned}$$

2.5-19 Recall $M(0)=1$ $M'(0)=E(x)=\mu$ $M''(0)=E(x^2)$

a.) $R'(0) = \dfrac{M'(0)}{M(0)} = \dfrac{\mu}{1} = \mu$

b.) $R''(0) = \dfrac{M(0)M''(0) - [M'(0)]^2}{[M(0)]^2}$

$= \dfrac{1 \cdot E(x^2) - \mu^2}{1^2}$

$= E(x^2) - \mu^2 = \sigma^2$



**2.5-20** **(a)**

$$\begin{aligned}
R(t) &= \ln(1-p+pe^t),\\
R'(t) &= \left[\frac{pe^t}{1-p+pe^t}\right]_{t=0} = p,\\
R''(t) &= \left[\frac{(1-p+pe^t)(pe^t)-(pe^t)(pe^t)}{(1-p+pe^t)^2}\right]_{t=0} = p(1-p);
\end{aligned}$$

**(b)**

$$\begin{aligned}
R(t) &= n\ln(1-p+pe^t),\\
R'(t) &= \left[\frac{npe^t}{1-p+pe^t}\right]_{t=0} = np,\\
R''(t) &= n\left[\frac{(1-p+pe^t)(pe^t)-(pe^t)(pe^t)}{(1-p+pe^t)^2}\right]_{t=0} = np(1-p);
\end{aligned}$$

**(c)**

$$\begin{aligned}
R(t) &= \ln p + t - \ln[1-(1-p)e^t],\\
R'(t) &= \left[1+\frac{(1-p)e^t}{1-(1-p)e^t}\right]_{t=0} = 1+\frac{1-p}{p} = \frac{1}{p},\\
R''(t) &= [(-1)\{1-(1-p)e^t\}^2\{-(1-p)e^t\}]_{t=0} = \frac{1-p}{p};
\end{aligned}$$

**(d)**

$$\begin{aligned}
R(t) &= r\left[\ln p + t - \ln\{1-(1-p)e^t\}\right],\\
R'(t) &= r\left[\frac{1}{1-(1-p)e^t}\right]_{t=0} = \frac{r}{p},\\
R''(t) &= r\left[(-1)\{1-(1-p)e^t\}^{-2}\{-(1-p)e^t\}\right]_{t=0} = \frac{r(1-p)}{p^2}.
\end{aligned}$$

**2.5-22** $(0.7)(0.7)(0.3) = 0.147$.