Let $u = y_2(1 + y_1^2/r)$. Then $y_2 = \dfrac{u}{1 + y_1^2/r}$ and $\dfrac{dy_2}{du} = \dfrac{1}{1 + y_1^2/r}$. So

$$\begin{aligned} g_1(y_1) &= \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1 + y_1^2/r)^{(r+1)/2}} \int_0^\infty \frac{1}{\Gamma[(r+1)/2]2^{(r+1)/2}} u^{(r+1)/2-1} e^{-u/2} \\ &= \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1 + y_1^2/r)^{(r+1)/2}}, \qquad -\infty < y_1 < \infty. \end{aligned}$$

5.3–18 Let $Y = X_1 + X_2 + \cdots + X_n$. Then Y is $N(800n, 100^2 n)$. Thus

$$\begin{aligned} P(Y \ge 10000) &= 0.90 \\ P\left(\frac{Y - 800n}{100\sqrt{n}} \ge \frac{10000 - 800n}{100\sqrt{n}}\right) &= 0.90 \\ -1.282 &= \frac{10000 - 800n}{100\sqrt{n}} \\ 800n - 128.2\sqrt{n} - 10000 &= 0. \end{aligned}$$

Either use the quadratic formula to solve for $\sqrt{n}$ or use Maple to solve for n. We find that $\sqrt{n} = 3.617$ or $n = 13.08$ so use $n = 14$ bulbs.

5.3–20 Note that $Y - X$ is $N(10000, 5000^2 + 6000^2)$. So the probability that B's total claims exceed those of A is

$$\begin{aligned} (0.80)(0.10) + (0.20)(0.10)P(Y - X > 0) &= 0.08 + 0.02\left[1 - \Phi\left(\frac{-10000}{7810.25}\right)\right] \\ &= 0.08 + 0.02(0.8997) = 0.098. \end{aligned}$$

5.4 The Central Limit Theorem

5.4–2 If

$$\begin{aligned} f(x) &= (3/2)x^2, \qquad -1 < x < 1, \\ E(X) &= \int_{-1}^{1} x(3/2)x^2\,dx = 0; \\ \mathrm{Var}(X) &= \int_{-1}^{1} (3/2)x^4\,dx = \left[\frac{3}{10}x^5\right]_{-1}^{1} = \frac{3}{5}. \end{aligned}$$

Thus

$$\begin{aligned} P(-0.3 \le Y \le 1.5) &= P\left(\frac{-0.3 - 0}{\sqrt{15(3/5)}} \le \frac{Y - 0}{\sqrt{15(3/5)}} \le \frac{1.5 - 0}{\sqrt{15(3/5)}}\right) \\ &\approx P(-0.10 \le Z \le 0.50) = 0.2313. \end{aligned}$$

5.4–4

$$\begin{aligned} P(39.75 \le \overline{X} \le 41.25) &= P\left(\frac{39.75 - 40}{\sqrt{(8/32)}} \le \frac{\overline{X} - 40}{\sqrt{(8/32)}} \le \frac{41.25 - 40}{\sqrt{(8/32)}}\right) \\ &\approx P(-0.50 \le Z \le 2.50) = 0.6853. \end{aligned}$$

5.4–6 (a)
$$\mu = \int_0^2 x(1-x/2)\,dx = \left[\frac{x^2}{2}-\frac{x^3}{6}\right]_0^2 = 2-\frac{4}{3}=\frac{2}{3};$$
$$\sigma^2 = \int_0^2 x^2(1-x/2)\,dx - \left(\frac{2}{3}\right)^2 = \left[\frac{x^3}{3}-\frac{x^4}{8}\right]_0^2 - \frac{4}{9} = \frac{2}{9}.$$

(b)
$$P\left(\frac{2}{3}\le \overline{X}\le\frac{5}{6}\right) = P\left(\frac{\frac{2}{3}-\frac{2}{3}}{\sqrt{\frac{2}{9}/18}}\le\frac{\overline{X}-\frac{2}{3}}{\sqrt{\frac{2}{9}/18}}\le\frac{\frac{5}{6}-\frac{2}{3}}{\sqrt{\frac{2}{9}/18}}\right) \approx P(0\le Z\le 1.5) = 0.4332.$$

5.4–8 (a) $E(\overline{X}) = \mu = 24.43;$

(b) $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n} = \frac{2.20}{30} = 0.0733;$

(c)
$$P(24.17\le\overline{X}\le 24.82) \approx P\left(\frac{24.17-24.43}{\sqrt{0.0733}}\le Z\le\frac{24.82-24.43}{\sqrt{0.0733}}\right) = P(-0.96\le Z<1.44) = 0.7566.$$

5.4–10 Using the normal approximation,
$$P(1.7\le Y\le 3.2) = P\left(\frac{1.7-2}{\sqrt{4/12}}\le\frac{Y-2}{\sqrt{4/12}}\le\frac{3.2-2}{\sqrt{4/12}}\right) \approx P(-0.52\le Z\le 2.078) = 0.6796.$$
Using the p.d.f. of Y,
$$\begin{aligned}P(1.7\le Y\le 3.2) &= \textstyle\int_{1.7}^{2}[(-1/2)y^3+2y^2-2y+(2/3)]\,dy \\ &\quad + \textstyle\int_2^3[(1/2)y^3-4y^2+10y-22/3]\,dy \\ &\quad + \textstyle\int_3^{3.2}[(-1/6)y^3+2y^2-8y+32/3]\,dy \\ &= [(-1/8)y^4+(2/3)y^3-y^2+(2/3)y]_{1.7}^2 \\ &\quad + [(1/8)y^4-(4/3)y^3+5y^2-(22/3)y]_2^3 \\ &\quad + [(-1/24)y^4+(2/3)y^3-4y^2+(32/3)y]_3^{3.2} \\ &= 0.1920+0.4583+0.0246 = 0.6749.\end{aligned}$$

5.4–12 The distribution of $\overline{X}$ is $N(2000, 500^2/25)$. Thus
$$P(\overline{X}>2050) = P\left(\frac{\overline{X}-2000}{500/5}>\frac{2050-2000}{500/5}\right)\approx 1-\Phi(0.50) = 0.3085.$$

5.4–14
$$\begin{aligned}E(X+Y) &= 30+50 = 80; \\ \mathrm{Var}(X+Y) &= \sigma_X^2+\sigma_Y^2+2\rho\sigma_X\sigma_Y \\ &= 52+64+28 = 144; \\ Z &= \sum_{i=1}^{25}(X_i+Y_i)\ \text{ in approximately }\ N(25\cdot 80, 25\cdot 144).\end{aligned}$$

$$\text{Thus} \quad P(1970 < Z < 2090) = P\left(\frac{1970-2000}{60} < \frac{Z-2000}{60} < \frac{2090-2000}{60}\right)$$
$$\approx \Phi(1.5) - \Phi(-0.5)$$
$$= 0.9332 - 0.3085 = 0.6247.$$

5.4–16 Let X_i equal the time between sales of ticket $i-1$ and i, for $i = 1, 2, \ldots, 10$. Each X_i has a gamma distribution with $\alpha = 3$, $\theta = 2$. $Y = \sum_{i=1}^{10} X_i$ has a gamma distribution with parameters $\alpha_Y = 30, \theta_Y = 2$. Thus

$$P(Y \le 60) = \int_0^{60} \frac{1}{\Gamma(30)2^{30}} y^{30-1} e^{-y/2}\, dy = 0.52428 \text{ using Maple.}$$

The normal approximation is given by

$$P\left(\frac{Y-60}{\sqrt{120}} \le \frac{60-60}{\sqrt{120}}\right) \approx \Phi(0) = 0.5000.$$

5.4–18 We are given that $Y = \sum_{i=1}^{20} X_i$ has mean 200 and variance 80. We want to find y so that

$$P(Y \ge y) < 0.20$$
$$P\left(\frac{Y-200}{\sqrt{80}} > \frac{y-200}{\sqrt{80}}\right) < 0.20;$$

We have that

$$\frac{y-200}{\sqrt{80}} = 0.842$$
$$y = 207.5 \uparrow 208 \text{ days.}$$

5.5 Approximations for Discrete Distributions

5.5–2 **(a)** $P(2 < X < 9) = 0.9532 - 0.0982 = 0.8550;$

(b)
$$P(2 < X < 9) = P\left(\frac{2.5-5}{2} \le \frac{X-25(0.2)}{\sqrt{25(0.2)(0.8)}} \le \frac{8.5-5}{2}\right)$$
$$\approx P(-1.25 \le Z \le 1.75)$$
$$= 0.8543.$$

5.5–4
$$P(35 \le X \le 40) \approx P\left(\frac{34.5-36}{3} \le Z \le \frac{40.5-36}{3}\right)$$
$$= P(-0.50 \le Z \le 1.50) = 0.6247.$$

5.5–6 $\mu_X = 84(0.7) = 58.8$, $\text{Var}(X) = 84(0.7)(0.3) = 17.64$,

$$P(X \le 52.5) \approx \Phi\left(\frac{52.5-58.8}{4.2}\right) = \Phi(-1.5) = 0.0668.$$

5.5–8 **(a)**
$$P(X < 20.857) = P\left(\frac{X-21.37}{0.4} < \frac{20.857-21.37}{0.4}\right)$$
$$= P(Z < -1.282) = 0.10.$$

(b) The distribution of Y is $b(100, 0.10)$. Thus

$$P(Y \le 5) = P\left(\frac{Y - 100(0.10)}{\sqrt{100(0.10)(0.90)}} \le \frac{5.5 - 10}{3}\right) \approx P(Z \le -1.50) = 0.0668.$$

(c) $$\begin{aligned} P(21.31 \le \overline{X} \le 21.39) &\approx P\left(\frac{21.31 - 21.37}{0.4/10} \le Z \le \frac{21.39 - 21.37}{0.4/10}\right) \\ &= P(-1.50 \le Z \le 0.50) = 0.6247. \end{aligned}$$

5.5–10 $$\begin{aligned} P(4776 \le X \le 4856) &\approx P\left(\frac{4775.5 - 4829}{\sqrt{4829}} \le Z \le \frac{4857.5 - 4829}{\sqrt{4829}}\right) \\ &= P(-0.77 \le Z \le 0.41) = 0.4385. \end{aligned}$$

5.5–12 The distribution of Y is $b(1000, 18/38)$. Thus

$$P(Y > 500) \approx P\left(Z \ge \frac{500.5 - 1000(18/38)}{\sqrt{1000(18/38)(20/38)}}\right) = P(Z \ge 1.698) = 0.0448.$$

5.5–14 **(a)** $E(X) = 100(0.1) = 10,\ \mathrm{Var}(X) = 9,$

$$\begin{aligned} P(11.5 < X < 14.5) &\approx \Phi\left(\frac{14.5 - 10}{3}\right) - \Phi\left(\frac{11.5 - 10}{3}\right) \\ &= \Phi(1.5) - \Phi(0.5) = 0.9332 - 0.6915 = 0.2417. \end{aligned}$$

(b) $P(X \le 14) - P(X \le 11) = 0.917 - 0.697 = 0.220;$

(c) $\displaystyle\sum_{x=12}^{14} \binom{100}{x} (0.1)^x (0.9)^{100-x} = 0.2244.$

5.5–16 **(a)** $E(Y) = 24(3.5) = 84,\ \mathrm{Var}(Y) = 24(35/12) = 70,$

$$P(Y \ge 85.5) \approx 1 - \Phi\left(\frac{85.5 - 84}{\sqrt{70}}\right) = 1 - \Phi(0.18) = 0.4286;$$

(b) $P(Y < 85.5) \approx 1 - 0.4286 = 0.5714;$

(c) $P(70.5 < Y < 86.5) \approx \Phi(0.30) - \Phi(-1.61) = 0.6179 - 0.0537 = 0.5642.$

5.5–18 **(a)**

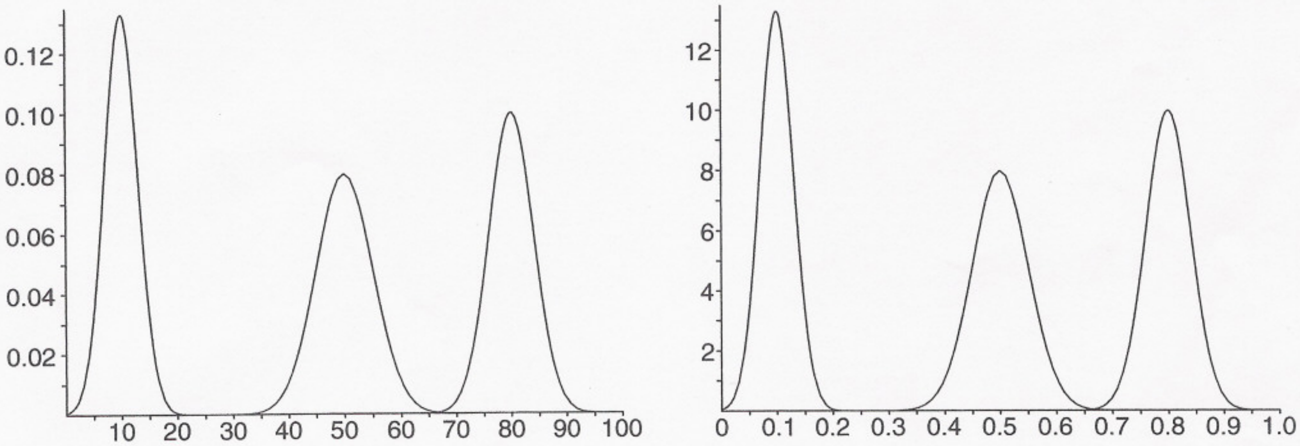

Figure 5.5–18: Normal approximations of the p.d.f.s of Y and $Y/100$, $p = 0.1, 0.5, 0.8$

(b) When $p = 0.1$,

$$P(-1.5 < Y - 10 < 1.5) \approx \Phi\left(\frac{1.5}{3}\right) - \Phi\left(\frac{-1.5}{3}\right) = 0.6915 - 0.3085 = 0.3830;$$

When $p = 0.5$,

$$P(-1.5 < Y - 50 < 1.5) \approx \Phi\left(\frac{1.5}{5}\right) - \Phi\left(\frac{-1.5}{5}\right) = 0.6179 - 0.3821 = 0.2358;$$

When $p = 0.8$,

$$P(-1.5 < Y - 80 < 1.5) \approx \Phi\left(\frac{1.5}{4}\right) - \Phi\left(\frac{-1.5}{4}\right) = 0.6462 - 0.3538 = 0.2924.$$

5.5–20

$$\begin{aligned} P(X > 35) &= P\left(\frac{X - 25}{5} > \frac{35.5 - 25}{5}\right) \\ &\approx 1 - \Phi(2.1) = 0.0179. \end{aligned}$$

Note that $P(X > 35) = 0.0225$ using Minitab.

5.5–22 **(a)** Y has a Poisson distribution with mean 30.

(b)

$$\begin{aligned} P(Y \le 25) &= P\left(\frac{Y - 30}{\sqrt{30}} \le \frac{25.5 - 30}{\sqrt{30}}\right) \\ &\approx \Phi(-0.8216) = 0.2057. \end{aligned}$$

Using Minitab, $P(Y \le 25) = 0.2084$.

6.2 Point Estimation

6.2–2 The likelihood function is

$$L(\theta) = \left[\frac{1}{2\pi\theta}\right]^{n/2} \exp\left[-\sum_{i=1}^{n}(x_i-\mu)^2/2\theta\right], \qquad 0<\theta<\infty.$$

The logarithm of the likelihood function is

$$\ln L(\theta) = -\frac{n}{2}(\ln 2\pi) - \frac{n}{2}(\ln\theta) - \frac{1}{2\theta}\sum_{i=1}^{n}(x_i-\mu)^2.$$

Setting the first derivative equal to zero and solving for θ yields

$$\begin{aligned}\frac{d\ln L(\theta)}{d\theta} &= -\frac{n}{2\theta} + \frac{1}{2\theta^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0\\ \theta &= \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2.\end{aligned}$$

Thus

$$\widehat{\theta} = \frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2.$$

To see that $\widehat{\theta}$ is an unbiased estimator of θ, note that

$$E(\widehat{\theta}) = E\left(\frac{\sigma^2}{n}\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma^2}\right) = \frac{\sigma^2}{n}\cdot n = \sigma^2,$$

since $(X_i-\mu)^2/\sigma^2$ is $\chi^2(1)$ and hence the expected value of each of the n summands is equal to 1.

6.2–4 **(a)** $\overline{x} = 394/7 = 56.2857$; $s^2 = 5452/97 = 56.2062$;

(b) $\widehat{\lambda} = \overline{x} = 394/7 = 56.2857$;

(c) Yes;

(d) $\overline{x}$ is better than s^2 because

$$\mathrm{Var}(\,\overline{X}\,) \approx \frac{56.2857}{98} = 0.5743 < 65.8956 = \frac{56.2857[2(56.2857*98)+97]}{98(97)} \approx \mathrm{Var}(S^2).$$

6.2–6 $\widehat{\theta}_1 = \widehat{\mu} = 33.4267$; $\widehat{\theta}_2 = \widehat{\sigma^2} = 5.0980$.

6.2–8 **(a)**

$$\begin{aligned} L(\theta) &= \left(\frac{1}{\theta^n}\right)\left(\prod_{i=1}^{n}x_i\right)^{1/\theta-1}, \qquad 0<\theta<\infty\\ \ln L(\theta) &= -n\ln\theta + \left(\frac{1}{\theta}-1\right)\ln\prod_{i=1}^{n}x_i\\ \frac{d\ln L(\theta)}{d\theta} &= \frac{-n}{\theta} - \frac{1}{\theta^2}\ln\prod_{i=1}^{n}x_i = 0\\ \widehat{\theta} &= -\frac{1}{n}\ln\prod_{i=1}^{n}X_i\\ &= -\frac{1}{n}\sum_{i=1}^{n}\ln X_i.\end{aligned}$$

(b) We first find $E(\ln X)$:

$$E(\ln X) = \int_0^1 \ln x (1/\theta) x^{1/\theta - 1}\, dx.$$

Using integration by parts, with $u = \ln x$ and $dv = (1/\theta)x^{1/\theta-1}dx$,

$$E(\ln X) = \lim_{a \to 0} \left[x^{1/\theta} \ln x - \theta x^{1/\theta} \right]_a^1 = -\theta.$$

Thus

$$E(\,\widehat{\theta}\,) = -\frac{1}{n} \sum_{i=1}^{n} (-\theta) = \theta.$$

6.2–10 **(a)** $\overline{x} = 1/p$ so $\widetilde{p} = 1/\overline{X} = n/\sum_{i=1}^{n} X_i$;

(b) $\widetilde{p}$ equals the number of successes, n, divided by the number of Bernoulli trials, $\sum_{i=1}^{n} X_i$;

(c) $20/252 = 0.0794$.

6.2–12 **(a)** $E(\,\overline{X}\,) = E(Y)/n = np/n = p$;

(b) $\text{Var}(\,\overline{X}\,) = \text{Var}(Y)/n^2 = np(1-p)/n^2 = p(1-p)/n$;

(c)
$$\begin{aligned} E[\overline{X}(1-\overline{X})/n] &= [E(\,\overline{X}\,) - E(\,\overline{X}^{\,2})]/n \\ &= \{p - [p^2 + p(1-p)/n]\}/n = [p(1-1/n) - p^2(1-1/n)]/n \\ &= (1-1/n)p(1-p)/n = (n-1)p(1-p)/n^2; \end{aligned}$$

(d) From part (c), the constant $c = 1/(n-1)$.

6.2–14 **(a)**
$$\begin{aligned} E(cS) &= E\left\{ \frac{c\sigma}{\sqrt{n-1}} \left[\frac{(n-1)S^2}{\sigma^2} \right]^{1/2} \right\} \\ &= \frac{c\sigma}{\sqrt{n-1}} \int_0^\infty \frac{v^{1/2} v^{(n-1)/2-1} e^{-v/2}}{\Gamma\left(\frac{n-1}{2}\right) 2^{(n-1)/2}}\, dv \\ &= \frac{c\sigma}{\sqrt{n-1}} \frac{\sqrt{2}\,\Gamma(n/2)}{\Gamma[(n-1)/2)]}, \end{aligned}$$

so $c = \dfrac{\sqrt{n-1}\,\Gamma[(n-1)/2]}{\sqrt{2}\,\Gamma(n/2)}$;

(b) When $n = 5$, $c = 8/(3\sqrt{2\pi}\,)$ and when $n = 6$, $c = 3\sqrt{5\pi}/(8\sqrt{2}\,)$.

(c)

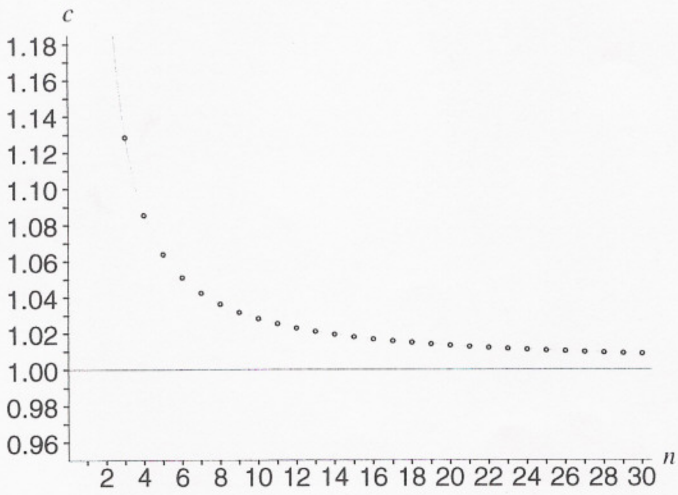

Figure 6.2–14: c as a function of n

We see that

$$\lim_{n\to\infty} c = 1.$$

6.2–16 $\overline{x} = \alpha\theta$, $v = \alpha\theta^2$ so that $\widetilde{\theta} = v/\overline{x}$, $\widetilde{\alpha} = \overline{x}^2/s^2$. For the given data, $\widetilde{\alpha} = 102.4990$, $\widetilde{\theta} = 0.0658$. Note that $\overline{x} = 6.74, v = 0.4432, s^2 = 0.4617$.

6.2–18 The experiment has a hypergeometric distribution with $n = 8$ and $N = 64$. From the sample, $\overline{x} = 1.4667$. Using this as an estimate for μ we have

$$1.4667 = 8\left(\frac{N_1}{64}\right) \quad \text{implies that} \quad \widetilde{N_1} = 11.73.$$

A guess for the value of N_1 is therefore 12.

6.3 Sufficient Statistics

6.3–2 The distribution of Y is Poisson with mean $n\lambda$. Thus, since $y = \Sigma\, x_i$,

$$\begin{aligned} P(X_1 = x_1,\ X_2 = x_2,\ \cdots,\ X_n = x_n \,|\, Y = y) &= \frac{(\lambda^{\Sigma x_i} e^{-n\lambda})/(x_1!\, x_2! \cdots x_n!)}{(n\lambda)^y\, e^{-n\lambda}/y!} \\ &= \frac{y!}{x_1!\ x_2! \cdots x_n!\, n^y}, \end{aligned}$$

which does not depend on λ.

6.3–4 (a) $f(x;\theta) = e^{(\theta-1)\ln x + \ln\theta}, \qquad 0 < x < 1, \qquad 0 < \theta < \infty;$

so $K(x) = \ln x$ and thus

$$Y = \sum_{i=1}^{n} \ln X_i = \ln(X_1 X_2 \cdots X_n)$$

is a sufficient statistic for θ.

(b)

$$\begin{aligned} L(\theta) &= \theta^n (x_1 x_2 \cdots x_n)^{\theta-1} \\ \ln L(\theta) &= n\ln\theta + (\theta-1)\ln(x_1 x_2 \cdots x_n) \\ \frac{d\ln L(\theta)}{d\theta} &= \frac{n}{\theta} + \ln(x_1 x_2 \cdots x_n) = 0. \end{aligned}$$

Hence

$$\widehat{\theta} = -n/\ln(X_1X_2\cdots X_n),$$

which is a function of Y.

(c) Since $\widehat{\theta}$ is a single valued function of Y with a single valued inverse, knowing the value of $\widehat{\theta}$ is equivalent to knowing the value of Y, and hence it is sufficient.

6.3–6 **(a)**

$$\begin{aligned} f(x_1, x_2, \ldots, x_n) &= \frac{(x_1x_2\cdots x_n)^{\alpha-1}e^{-\Sigma x_i/\theta}}{[\Gamma(\alpha)]^n\theta^{\alpha n}} \\ &= \left(\frac{e^{-\Sigma x_i/\theta}}{\theta^{\alpha n}}\right)\left(\frac{(x_1x_2\cdots x_n)^{\alpha-1}}{[\Gamma(\alpha)]^n}\right). \end{aligned}$$

The second factor is free of θ. The first factor is a function of the x_is through $\sum_{i=1}^n x_i$ only, so $\sum_{i=1}^n x_i$ is a sufficient statistic for θ.

(b)

$$\begin{aligned} \ln L(\theta) &= \ln(x_1x_2\cdots x_n)^{\alpha-1} - \textstyle\sum_{i=1}^n x_i/\theta - \ln[\Gamma(\alpha)]^n - \alpha n\ln\theta \\ \frac{d\ln L(\theta)}{d\theta} &= \textstyle\sum_{i=1}^n x_i/\theta^2 - \alpha\, n/\theta = 0 \\ \alpha\, n\theta &= \textstyle\sum_{i=1}^n x_i \\ \widehat{\theta} &= \frac{1}{\alpha\, n}\sum_{i=1}^n X_i. \end{aligned}$$

$Y = \displaystyle\sum_{i=1}^n X_i$ has a gamma distribution with parameters $\alpha\, n$ and θ. Hence

$$E(\widehat{\theta}) = \frac{1}{\alpha\, n}(\alpha\, n\,\theta) = \theta.$$

6.3–8

$$E(e^{tZ}) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi\theta}}\right)^n e^{-\Sigma x_i^2/(2\theta)}\cdot e^{t\Sigma a_ix_i/\Sigma x_i}dx_1dx_2\ldots dx_n.$$

Let $x_i/\sqrt{\theta} = y_i,\ i = 1, 2, \ldots, n$. The Jacobian is $(\sqrt{\theta})^n$. Hence

$$\begin{aligned} E(e^{tZ}) &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}(\sqrt{\theta})^n\left(\frac{1}{\sqrt{2\pi\theta}}\right)^n e^{-\Sigma y_i^2/2}\cdot e^{t\Sigma a_iy_i/\Sigma y_i}dy_1dy_2\ldots dy_n \\ &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\right)^n e^{-\Sigma y_i^2/2}\cdot e^{t\Sigma a_iy_i/\Sigma y_i}dy_1dy_2\ldots dy_n \end{aligned}$$

which is free of θ. Since the distribution of Z is free of θ, Z and $Y = \sum_{i=1}^n X_i^2$, the sufficient statistics, are independent.

6.4 Confidence Intervals for Means

6.4–2 **(a)** [77.272, 92.728]; **(b)** [79.12, 90.88]; **(c)** [80.065, 89.935]; **(d)** [81.154, 88.846].

6.4–4 **(a)** $\overline{x} = 56.8$;

(b) $\left[56.8 - 1.96(2/\sqrt{10}),\ 56.8 + 1.96(2/\sqrt{10})\right] = [55.56,\ 58.04]$;

(c) $P(X < 52) = P\left(Z < \dfrac{52 - 56.8}{2}\right) = P(Z < -2.4) = 0.0082.$

6.4–6 $\left[11.95 - 1.96\left(\frac{11.80}{\sqrt{37}}\right),\ 11.95 + 1.96\left(\frac{11.80}{\sqrt{37}}\right)\right] = [8.15,\ 15.75].$

If more extensive t-tables are available or if a computer program is used, we have

$$\left[11.95 - 2.028\left(\frac{11.80}{\sqrt{37}}\right),\ 11.95 + 2.028\left(\frac{11.80}{\sqrt{37}}\right)\right] = [8.016,\ 15.884].$$

6.4–8 **(a)** $\overline{x} = 46.42$;
(b) $46.72 \pm 2.132s/\sqrt{5}$ or $[40.26,\ 52.58]$.

6.4–10 $\left[21.45 - 1.314\left(\frac{0.31}{\sqrt{28}}\right), \infty\right) = [21.373,\ \infty)$.

6.4–12 **(a)** $\overline{x} = 3.580$;
(b) $s = 0.512$;
(c) $[0,\ 3.580 + 1.833(0.512/\sqrt{10}\,] = [0,\ 3.877]$.

6.4–14 **(a)** $\overline{x} = 245.80$, $s = 23.64$, so a 95% confidence interval for μ is

$$[245.80 - 2.145(23.64)/\sqrt{15}\,,\ 245.80 + 2.145(23.64)/\sqrt{15}\,] = [232.707,\ 258.893];$$

(b)

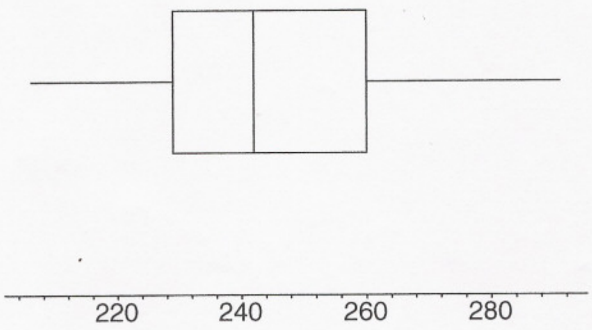

Figure 6.4–14: Box-and-whisker diagram of signals from detectors

(c) The standard deviation is quite large.

6.4–16 **(a)** $(\overline{x} + 1.96\sigma/\sqrt{5}\,) - (\overline{x} - 1.96\sigma/\sqrt{5}\,) = 3.92\sigma/\sqrt{5} = 1.753\sigma$;
(b) $(\overline{x} + 2.776s/\sqrt{5}\,) - (\overline{x} - 2.776s/\sqrt{5}\,) = 5.552s/\sqrt{5}$.

From Exercise 6.2–14 with $n = 5$, $E(S) = \frac{\sqrt{2}\,\Gamma(5/2)\sigma}{\sqrt{4}\,\Gamma(4/2)} = \frac{3\sqrt{\pi}\sigma}{2^{5/2}} = 0.94\sigma$, so that $E[5.552S/\sqrt{5}\,] = 2.334\sigma$.

6.4–18 $6.05 \pm 2.576(0.02)/\sqrt{1219}$ or $[6.049,\ 6.051]$.

6.4–20 **(a)** $\overline{x} = 4.483$, $s^2 = 0.1719$, $s = 0.4146$;
(b) $[4.483 - 1.714(0.4146)/\sqrt{24}),\ \infty) = [4.338,\ \infty)$;