Lab 7: The Normal 2-Sample Location Problem

OBJECTIVES: We introduce t-tests and corresponding confidence intervals for drawing inferences about the difference in means of two normal populations from which independent samples have been drawn. We apply these procedures to two problems:

(i) deciding whether or not two methods for determining the atomic weight of carbon differ, and

(ii) deciding whether or not babies of mothers who smoke have lower birth weights than babies of mothers who do not smoke.

ASSUMPTIONS: We assume that

$$X_1, \ldots, X_m \sim N(\mu_1, \sigma_1^2)$$

$$Y_1, \ldots, Y_n \sim N(\mu_2, \sigma_2^2)$$

and that $X_i \neq Y_j$ are mutually independent. We also assume that μ_1, μ_2, σ_1^2, and σ_2^2 are unknown. We <u>do not</u> assume that $\sigma_1^2 = \sigma_2^2$. Let $\Delta = \mu_1 - \mu_2$ denote the difference in population means, sometimes called the *shift parameter*.

PROCEDURES: Because we do not assume equality of variances, we emphasize the use of Welch's approximate t-test (the book does not give this test a name, but the Welch's t-test is what it is generally referred to as). Let $\bar{x}$, $\bar{y}$, and s_x^2, s_y^2 denote the sample means and sample variances for the two samples. Also, let m and n denote the sample size for the two samples. Then the two-sample t-statistic is

$$t_W = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_x^2}{m} + \frac{s_y^2}{n}}}$$

The distribution of this Welch's test statistic is approximately a t-distribution with the following number of degrees of freedom:

$$df = \frac{(s_x^2/m + s_y^2/n)^2}{\frac{(s_x^2/m)^2}{m-1} + \frac{(s_y^2/n)^2}{n-1}}$$

Remarks: When $m = n$ (i.e. when the sample sizes are equal), t_W equals Student's 2-sample t-statistic. The Welch test typically uses fewer degrees of freedom than the Student test.

A $(1-\alpha)$-level confidence interval for $\Delta = \mu_1 - \mu_2$ is

$$(\bar{x} - \bar{y}) \pm t_{(1-\alpha/2),df} \sqrt{\frac{s_x^2}{m} + \frac{s_y^2}{n}}$$

where $t_{(1-\alpha/2),df}$ is the *critical value.*

DIRECTIONS: Follow the instructions below, answering all questions. Write out the answers neatly and turn them in to the lab instructor.

1. Consider the following data set consisting of atomic weights of carbon found by 2 different methods:

Method A	Method B
12.0072	11.9853
12.0064	11.9949
12.0054	11.9985
12.0016	12.0061
12.0077	

Hypotheses: Let μ_A denote the population mean for method A, and μ_B the population mean for method B. We want to test $H_0 : \mu_A = \mu_B$ vs. $H_1 : \mu_A \neq \mu_B$, and we note that this is equivalent to testing: $H_0 : \Delta = 0$ vs. $H_1 : \Delta \neq 0$ where $\Delta = \mu_A - \mu_B$.

a. Create 2 columns titled WEIGHT and METHOD$. Before entering the weights in the WEIGHT column, make sure that the number of decimal places in the editor is at least 4. If it is not, then this can be changed by going to "Edit/Preferences" and changing the number of decimal places under the editor menu. Enter the five weights determined by method A in the WEIGHT column, followed by the four weights determined by method B. METHOD$ is a grouping variable for WEIGHT. For each number in the WEIGHT column, enter A or B in the METHOD$ column, whichever is appropriate.

b. Calculate and record the summary stastistics for each of the two samples. Use the "By Groups" option under the "Data" menu. Select METHOD$ as the "By Groups" variable and then "Weight" as the variable when you go to the "Stats" menu.

c. Construct normal probability plots for each sample. Before doing this, make sure the "Append Graphs" option is turned on under the "Window/ Graph Placement" menu. Do the samples appear to be normal? Do the samples appear to be symmetric? Can we learn very much with such a small sample size?

d. Calculate and compare the sample variances. Does it seem plausible that $\sigma_A^2 = \sigma_B^2$?

2. Now, turn "By Groups" off. Perform Welch's approximate t-test using a 2-sided significance level of $\alpha = 0.05$. To do this, select "t-test" under the "Stats" option of the "Stats" menu. Select the "independent" option in the lower right corner of the window. Then select WEIGHT from the variables box and METHOD$ from the group box. SYSTAT will then perform two t-tests. The *pooled variance* t-test is Students', whereas the *separate variance* t-test is Welch's.

a. What is t_W?

b. What is df?

c. What is the significance probability?

d. Should we reject H_0?

e. What would we have concluded had we used Students' test instead of Welch's?

3. Calculate the 2-sided 95% confidence interval for Δ. To determine the critical value, $t_{0.975,df}$, use the TIF(α,df) function under the "Math" menu. In order to do this, let TVAL=TIF(α,df). Note, you are looking for a 2-sided c.i., so choose the α appropriately. Also, when looking at the formula on the first page, be sure to note that $t_{(1-\alpha/2),df}$ is the critical value, not the value of Welch's test statistic that was asked for in part a of question 2.

4. What can be inferred about the two methods for determining the atomic weight of carbon?

5. The second data set we will consider today is data on the birth weights of smoking and non-smoking mothers. It is believed that babies of nonsmoking mothers tend to have higher birth weights than babies of smoking mothers.

   DATA: The data are from a study of the birth weights in pounds of the children of 40 mothers who smoked at least one pack a day during pregnancy and 39 mothers who did not smoke at all.

Children of Nonsmoking Mothers

8.3	7.9	9.6	7.1	6.8	10.2	7.3	8.8	8.0	9.5
5.9	10.1	8.2	8.7	9.6	12.3	8.1	7.3	7.8	6.6
9.1	7.4	6.8	7.5	8.2	6.6	7.9	8.4	8.9	10.4
9.0	7.5	8.2	8.7	7.0	10.8	9.9	8.8	12.3	

Children of Smoking Mothers

8.1	6.5	7.3	6.8	7.9	8.4	6.2	7.8	9.1	6.7
8.8	7.5	7.0	7.3	9.6	5.6	8.0	6.9	7.1	7.9
10.3	7.4	4.9	7.3	8.1	6.2	9.9	5.7	8.6	7.4
8.2	10.8	6.8	7.4	8.9	5.9	7.2	7.9	8.0	6.6

Hypotheses: Let μ_{NS} denote the population mean for the nonsmokers, and let μ_S denote the population mean for the smokers. We want to test $H_0 : \mu_{NS} \leq \mu_S$ vs. $H_1 : \mu_{NS} > \mu_S$. Note that this is equivalent to testing $H_0 : \Delta \leq 0$ vs. $H_1 : \Delta > 0$, where $\Delta = \mu_{NS} - \mu_S$.

a. Create a column titled WEIGHT. Enter the weights of the babies of *nonsmoking* mothers into the WEIGHT columns, followed by the weights of the babies of the smoking mothers. Next, we want to create a grouping variable CIG$. To do this, go to the command window and type:

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IF CASE<40 THEN LET CIG$='NO'
IF CASE>39 THEN LET CIG$='YES'
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b. Calculate and record summary statistics for each sample. Again, you will need to turn on the "By Groups" option under the "Data" menu.

c. Construct box plots and normal probability plots for each sample. Do the samples appear to be normal? Symmetric?

d. Calculate and compare the sample variances. Does it seem plausible that $\sigma_{NS}^2 = \sigma_S^2$?

6. Turn off "By Groups". Perform Welch's approximate t-test using a 1-sided significance level of $\alpha = 0.05$.

    a. What is t_W?

    b. What is df?

    c. What is the significance probability?

    d. Should we reject H_0?

7. Calculate the 2-sided 0.95-level confidence interval for Δ. Don't forget that the degrees of freedom are different for this data set, hence you will have to recalculate $t_{0.975,df}$ using the new df. NOTE: You will need to use the formula on the first page of the lab to calculate the confidence interval.

8. What can be inferred about the birth weights of smoking and nonsmoking mothers?