STAT421: Practice Final Solutions

May 5, 1999

1. 50 points Consider an MA(2) process Z_t defined as follows:

$$\tilde{Z}_t = a_t + (7/6)a_{t-1} + (1/3)a_{t-2},$$

where

$$\tilde{Z}_t = Z_t - 10,$$

and a_t is an i.i.d. $N(0, 25)$ sequence.
(a) Verify that Z_t is invertible.
(b) Compute $\mu = E[Z_t]$, $\sigma_Z^2 = \text{Var}[Z_t]$, and the autocorrelation function ρ_h, $h = 0, 1, \ldots$.
(c) Suppose you are given that $a_{-1} = 6$ and $a_0 = -9$. Compute forecast values for Z_1 and Z_2.
(d) Compute the forecast variance $\text{Var}[Z_1|a_0, a_{-1}]$ and $\text{Var}[Z_2|a_0, a_{-1}]$ for your results in part (c).
(e) What are the forecasts and forecast variances for Z_t, $t > 2$ given a_0 and a_{-1}?

Solutions: To check invertibility, we compute the roots of the characteristic equation:

$$1 + (7/6)v + (1/3)v^2 = 0.$$

The l.h.s. can be factored:

$$(1 + (2/3)v)(1 + (1/2)v) = 0.$$

The roots are

$$v_1 = -3/2 \quad \& \quad v_2 = -2.$$

These are clearly outside the unit circle, so the process is invertible.

Clearly $E[\tilde{Z}_t] = 0$ and $E[Z_t] = 10$. For $\text{Var}[Z_t] = \text{Var}[\tilde{Z}_t]$ we have

$$\begin{aligned}
\text{Var}[\tilde{Z}_t] &= E[\tilde{Z}_t^2] \\
&= E\left[(a_t + (7/6)a_{t-1} + (1/3)a_{t-2})^2\right] \\
&= E[a_t^2] + (7/6)^2 E[a_{t-1}^2] + (1/3)^2 E[a_{t-2}^2] \\
&= (1 + 49/36 + 1/9)\,\sigma_a^2 \\
&= (89/36) \cdot 25 = 2225/36 = 61.81.
\end{aligned}$$

Turning now to the ACF ρ_h, since it is an MA(2) we have

$$\rho_h = 0, \quad |h| > 2.$$

Of course, $\rho_0 = 1$, so we need only compute ρ_1 and ρ_2. We can pretend $\sigma_a^2 = 1$ and use the MA equation. Multiplying through by $\tilde{Z}_{t-1}$ and taking expectations gives

$$\begin{aligned}
E[\tilde{Z}_t \tilde{Z}_{t-1}] &= E\left[(a_t + (7/6)a_{t-1} + (1/3)a_{t-2})\,(a_{t-1} + (7/6)a_{t-2} + (1/3)a_{t-3})\right] \\
&= (7/6) \cdot 1 + (1/3) \cdot (7/6) \\
&= 28/18 = 14/9,
\end{aligned}$$

so

$$\rho_1 \;=\; (14/9)/(89/36) \;=\; 56/89 \;=\; 0.6292135.$$

For ρ_2 we have

$$\begin{aligned} E[\tilde{Z}_t\tilde{Z}_{t-2}] &= E\left[(a_t + (7/6)a_{t-1} + (1/3)a_{t-2})\,(a_{t-2} + (7/6)a_{t-3} + (1/3)a_{t-4})\right] \\ &= 1/3. \end{aligned}$$

and

$$\rho_2 \;=\; (1/3)/(89/36) \;=\; 12/89 \;=\; 0.1348315.$$

Turning now to part (c), we have in general by linearity of conditional expectation that

$$\begin{aligned} E[\tilde{Z}_t|a_0, a_{-1}] &= E[a_t + (7/6)a_{t-1} + (1/3)a_{t-2}|a_0, a_{-1}] \\ &= E[a_t|a_0, a_{-1}] \;+\; (7/6)E[a_{t-1}|a_0, a_{-1}] \;+\; (1/3)E[a_{t-2}|a_0, a_{-1}], \end{aligned}$$

and of course since a_t is a white noise

$$E[a_t|a_0, a_{-1}] \;=\; \begin{cases} a_t & \text{if } t = 0 \text{ or } t = -1 \\ 0 & \text{otherwise.} \end{cases}$$

Thus,

$$E[\tilde{Z}_1|a_0, a_{-1}] \;=\; 0 \;+\; (7/6)a_0 \;+\; (1/3)a_{-1} \;=\; (7/6)\cdot(-9) \;+\; (1/3)\cdot 6 \;=\; -8.5,$$

so

$$E[Z_1|a_0, a_{-1}] \;=\; 10 \;+\; (-8.5) \;=\; 1.5.$$

Continuing

$$E[\tilde{Z}_2|a_0, a_{-1}] \;=\; 0 \;+\; (7/6)\cdot 0 \;+\; (1/3)a_0 \;=\; -3,$$

and

$$E[Z_2|a_0, a_{-1}] \;=\; 10 \;+\; (-3) \;=\; 7.$$

For part (d), since the a_t's are a mutually independent mean 0 stationary white noise process, we have

$$\mathrm{Var}[a_t|a_0, a_{-1}] \;=\; E[a_t^2|a_0, a_{-1}] \;=\; \begin{cases} \sigma_a^2 & \text{if } t/ne0 \text{ and } t/ne-1, \\ 0 & \text{if } t = 0 \text{ or } t = -1. \end{cases}$$

and

$$\mathrm{Cov}[a_t, a_s|a_0, a_{-1}] \;=\; 0, \quad \forall s \neq t.$$

This is clearly true if neither s nor t is 0 or -1, and if one or the other is one of the given a's then it is treated as a constant which has 0 covariance with anything. Thus,

$$\begin{aligned} \mathrm{Var}[Z_1|a_0, a_{-1}] &= \mathrm{Var}[\tilde{Z}_1|a_0, a_{-1}] \\ &= \mathrm{Var}[a_1|a_0, a_{-1}] \;+\; (7/6)^2\mathrm{Var}[a_0|a_0, a_{-1}] \;+\; (1/3)^2\mathrm{Var}[a_{-1}|a_0, a_{-1}] \\ &= \mathrm{Var}[a_1|a_0, a_{-1}] \;+\; 0 \;+\; 0 \\ &= \sigma_a^2. \end{aligned}$$

Also,

$$\begin{aligned}
\mathrm{Var}[Z_2|a_0, a_{-1}] &= \mathrm{Var}[a_2|a_0, a_{-1}] + (7/6)^2\mathrm{Var}[a_1|a_0, a_{-1}] + (1/3)^2\mathrm{Var}[a_0|a_0, a_{-1}] \\
&= \sigma_a^2 + (7/6)^2\sigma_a^2 + 0 \\
(85/36)\sigma_a^2.
\end{aligned}$$

For part (e), as already noted above Z_t is independent of a_0 and a_{-1} once $t > 1$, since then Z_t is a linear combination of 3 a_t's which are disjoint from a_0 and a_{-1}. Thus, the conditional mean and variance will be the same as the unconditional mean and variance already given in part (b):

$$\begin{aligned}
E[Z_t|a_0, a_{-1}] &= 10 \\
\mathrm{Var}[Z_t|a_0, a_{-1}] &= 61.81.
\end{aligned}$$

2. 30 points A statistician buys a new car and records his gas mileage on the first 118 fillups (REAL DATA!). Figure 1 on the next page shows a time series plot, sample ACF, and sample PACF for the original series. Figure 2 shows the analogous plots for the series when the first 40 observations are deleted (only the last 78 observations are kept). The objective is to predict mileage for fillups 119, 120 , ...
(a) For each series (full series, last 78 observations), describe a tentative ARMA model identification based on the information provided. Justify your answers.
(b) Would you recommend using the full series or just the last 78 observations for model fitting and then forecasting? Justify your answer.

Solution. Although the level of the series appears to be roughly constant at roughly 16 mpg, the first 40 or so observations seem to be much less variable than the last 78 observations, so we would question as to whether the original data come from a stationary process. When looking at the top plot in Figure 2, however, the level and variability look fairly constant. Clearly the last 78 observations are more relevant to the observations just beyond what we have observed, so based on this I would advocate using only the last 78 observations to build a model. This seems like an adequate number of observations to build a low order ARMA model.

Also, looking at the ACF and PACF, we see that the full 118 observations suggests an initial identification of either an MA(4) or an AR(2), whereas the last 78 observations suggest an MA(3) or an AR(1). If the AR(1) model were to proved adequate after fitting, this would be far simpler than any of the other models. So it appears that using only the last 78 observations will permit a more parsimonious model as well.

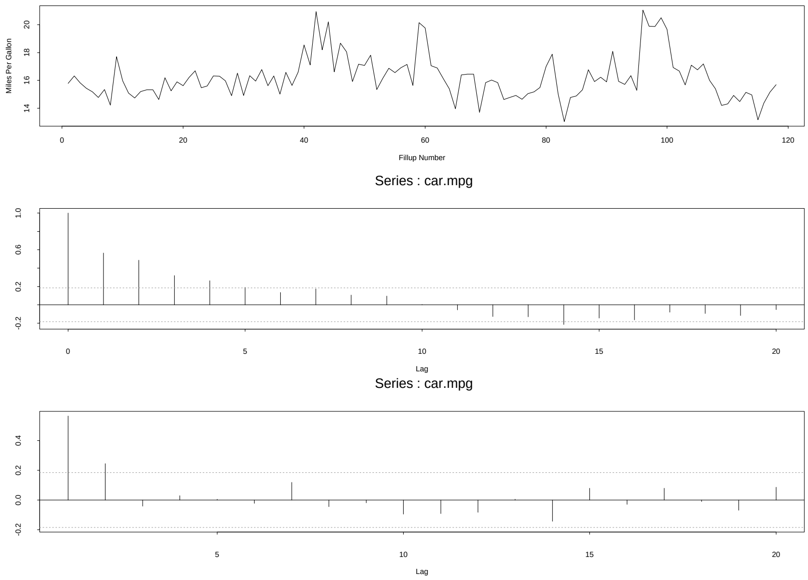

Figure 1: Time Series Plot, sample ACF, and sample PACF for the full series in Problem 2.

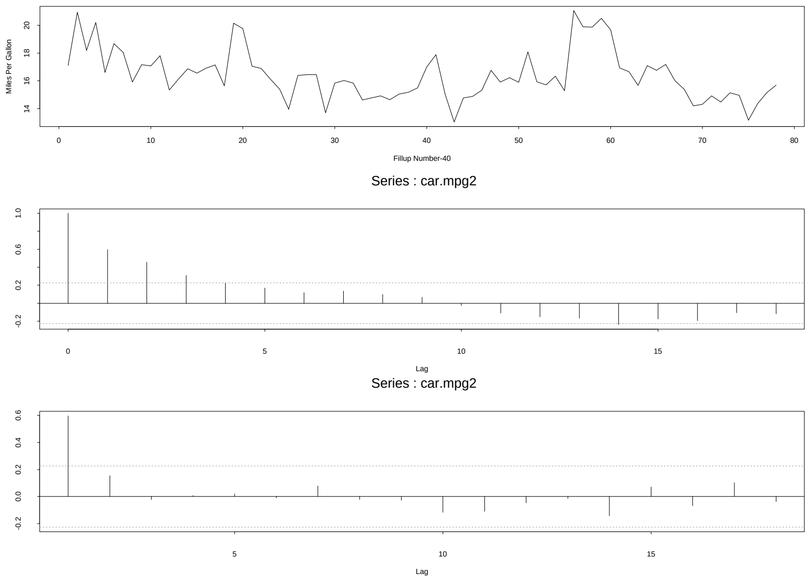

Figure 2: Time Series Plot, sample ACF, and sample PACF for the last 78 observations of the series in Problem 2.

(3) 36 points (Forecasts for MA processes.) Consider an MA(1) process

$$Z_t = a_t - \theta a_{t-1}$$

where the a_t's are i.i.d. $N(0, \sigma_a^2)$.
(a) Show that $E[Z_1|a_0]$ is $-\theta a_0$.
(b) Show that $E[Z_2|Z_1, a_0]$ is $-\theta[Z_1 - \theta a_0]$.
(c) What is $E[Z_3|Z_2, Z_1, a_0)$?
(d) Show how one can recursively compute $E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0)$.
(e) Of course, in practice we won't observe a_0. Show that $E[Z_t|Z_1, \ldots, Z_{t-1}]$ is given by

$$E[Z_t|Z_1, \ldots, Z_{t-1}] = \int E\left[Z_1, \ldots Z_{t-1}, a_0\right] \frac{1}{\sqrt{2\pi\sigma_a^2}} \exp\left[-a_0^2/(2\sigma_a^2)\right] da_0.$$

(f) Describe an algorithm for computing the forecasts for Z_{n+1} ,Z_{n+2}, …, $Z_{n+\ell}$ given data $(z_1, \ldots, z_n)$ modeled by the MA(1) process above.

Solutions: Just as in part (c) of Problem 1,

$$\begin{aligned} E[Z_1|a_0] &= E[a_1|a_0] - \theta E[a_0|a_0] \\ &= 0 - \theta a_0. \end{aligned}$$

Now for part (b), we have similarly

$$E[Z_2|Z_1, a_0] = E[a_2|Z_1, a_0] - \theta E[a_1|Z_1, a_0].$$

Now a_2 is independent of Z_1 and a_0, so

$$E[a_2|Z_1, a_0] = E[a_2] = 0.$$

Also, a_1 is determined exactly if we know Z_1 and a_0 since

$$a_1 = Z_1 + \theta a_0.$$

Thus,

$$E[Z_2|Z_1, a_0] = -\theta\,(Z_1 + \theta a_0)$$

Looks like there is an incorrect sign in the statement of the problem.

Turning to part (c),

$$E[Z_3|Z_2, Z_1, a_0] = E[a_3|Z_2, Z_1, a_0] - \theta E[a_2|Z_2, Z_1, a_0].$$

Again, a_3 is independent of (Z_2, Z_1, a_0) so

$$E[a_3|Z_2, Z_1, a_0] = 0.$$

From (Z_2, Z_1, a_0) we can perfectly determine a_2, viz.

$$\begin{aligned} a_2 &= Z_2 + \theta a_1 \\ &= Z_2 + \theta(Z_1 + \theta a_0). \end{aligned}$$

Thus,

$$\begin{aligned} E[Z_3|Z_2, Z_1, a_0] &= -\theta\left[Z_2 + \theta(Z_1 + \theta a_0)\right] \\ &= -\left[\theta Z_2 + \theta^2 Z_1 + \theta^3 a_0\right]. \end{aligned}$$

It looks like in general,

$$E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0] = -\left[\theta Z_{t-1} + \theta^2 Z_{t-2} + \cdots + \theta^{t-1} Z_1 + \theta^t a_0\right]. \quad (1)$$

Clearly

$$E[a_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0] = 0,$$

by the same independence argument as above, and a_{t-1} can be determined exactly from $(Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0)$ via

$$\begin{aligned} a_{t-1} &= Z_{t-1} + \theta a_{t-2} \\ &= Z_{t-1} + \theta Z_{t-2} + \theta^2 a_{t-3} \\ &= Z_{t-1} + \theta Z_{t-2} + \theta^2 Z_{t-3} + \theta^3 a_{t-4} \\ &= Z_{t-1} + \theta Z_{t-2} + \theta^2 Z_{t-3} + \theta^3 Z_{t-4} + \cdots + \theta^{t-2} Z_1 + \theta^{t-1} a_0. \end{aligned}$$

One can use an induction argument for complete mathematical rigor. So

$$\begin{aligned} &E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0] \\ &\quad= E[a_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0] - \theta E[a_{t-1}|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0] \\ &\quad= 0 - \theta\left[Z_{t-1} + \theta Z_{t-2} + \theta^2 Z_{t-3} + \theta^3 Z_{t-4} + \cdots + \theta^{t-2} Z_1 + \theta^{t-1} a_0\right] \end{aligned}$$

which proves (1).

How can this be used for a recursive algorithm? Letting $\hat{Z}_t$ denote $E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0]$, we have

$$\hat{Z}_{t-1} = -\left[\theta Z_{t-2} + \theta^2 Z_{t-3} + \theta^3 Z_{t-4} + \cdots + \theta^{t-2} Z_1 + \theta^{t-1} a_0\right]$$

so from (1)

$$\hat{Z}_t = -\theta(Z_{t-1} - \theta \hat{Z}_{t-1}).$$

This gives a recursive algorithm.

There seems to be a typo in part (d). Maybe it should read

$$\begin{aligned} &E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1] \\ &\quad= \int E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1, a_0] f(a_0)\, da_0 \end{aligned} \quad (2)$$

and then of course

$$f(a_0) \;=\; \frac{1}{\sqrt{2\pi}\sigma_a}\exp\left[-\frac{a_0^2}{2\sigma_a^2}\right]$$

is the $N(0, \sigma_a^2)$ density for a_0. To attempt a verification of (2), note that by definition of the conditional mean

$$\begin{aligned}&E[Z_t|Z_{t-1} = z_{t-1}, Z_{t-2} = z_{t-2}, \ldots, Z_1 = z_1, a_0] \\ &\quad= \int z_t f(z_t|z_{t-1}, z_{t-2}, \ldots, z_1, a_0)\, dz_t.\end{aligned}$$

and

$$\begin{aligned}&E[Z_t|Z_{t-1} = z_{t-1}, Z_{t-2} = z_{t-2}, \ldots, Z_1 = z_1] \\ &\quad= \int z_t f(z_t|z_{t-1}, z_{t-2}, \ldots, z_1)\, dz_t.\end{aligned}$$

Now by definition of conditional density:

$$f(z_t|z_{t-1}, z_{t-2}, \ldots, z_1) \quad=\quad \frac{f(z_t, z_{t-1}, z_{t-2}, \ldots, z_1)}{f(z_{t-1}, z_{t-2}, \ldots, z_1)}$$

and

$$f(z_t, z_{t-1}, z_{t-2}, \ldots, z_1) \;=\; \int f(z_t, z_{t-1}, z_{t-2}, \ldots, z_1, a_0)\, da_0.$$

But I don't see how we can get to

$$f(z_t|z_{t-1}, z_{t-2}, \ldots, z_1, a_0) \quad=\quad \frac{f(z_t, z_{t-1}, z_{t-2}, \ldots, z_1, a_0)}{f(z_{t-1}, z_{t-2}, \ldots, z_1, a_0)}$$

since this would require integrating over a_0 in *both* the numerator and denominator. There seems to be no way to get to (2). It seems Dr. Cox may have screwed up this problem worse than it appears on first glance. I hope there aren't such screw ups on the real final.

Well, let's see what we can salvage. Our argument above in fact shows that

$$\begin{aligned}Z_t &= a_t - \theta a_{t-1} \\ &= a_t - \left[\theta Z_{t-1} + \theta^2 Z_{t-2} + \cdots + \theta^{t-1} Z_1 + \theta^t a_0\right].\end{aligned}$$

Thus,

$$\begin{aligned}&E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1] \\ &\quad= \; -\left\{\theta Z_{t-1} + \theta^2 Z_{t-2} + \cdots + \theta^{t-1} Z_1 + \theta^t E[a_0|Z_{t-1}, Z_{t-2}, \ldots, Z_1]\right\} \qquad (3)\end{aligned}$$

Since we only have 3 hours (for the real final), we can't hope to correct all of Dr. Cox's mistakes. However, we can note that as long as t is relatively large and θ is less than 1 in magnitude, then the contribution from the term

$$\theta^t E[a_0|Z_{t-1}, Z_{t-2}, \ldots, Z_1]$$

decreases exponentially so is probably not too large. Therefore,

$$\begin{aligned} E[Z_t|Z_{t-1}, Z_{t-2}, \ldots, Z_1] &\doteq \hat{\hat{Z}}_t \\ &= -\left\{\theta Z_{t-1} + \theta^2 Z_{t-2} + \cdots + \theta^{t-1} Z_1\right\}, \end{aligned}$$

and that $\hat{\hat{Z}}_t$ can be computed by the same recursion as we found in part (d), namely

$$\hat{\hat{Z}}_t = -\theta(Z_{t-1} - \theta\hat{\hat{Z}}_{t-1}).$$

Proceeding to part (f), of course when we forecast Z_{t+1} given $Z_{t-1}, Z_{t-2}, \ldots, Z_1$ we just use the mean of the process because Z_{t+1} is independent of $Z_{t-1}, Z_{t-2}, \ldots, Z_1$. It is not clear if we are to assume a mean 0 process or not, so here's the algorithm for computing *approximately* the forecasts:

(1) Compute $\bar{z}$, the mean of the observed data. Compute $\tilde{z}_t = z_t - \bar{z}$ for $1 \leq t \leq n$.

(2) Let $\hat{\tilde{z}}_1 = \tilde{z}_1$, and for $t = 2, \ldots, n+1$, let

$$\hat{\tilde{z}}_t = -\theta(\tilde{z}_{t-1} - \theta\hat{\tilde{z}}_{t-1}).$$

This gives $\hat{\tilde{z}}_{n+1}$, the forecast for $\tilde{Z}_{n+1}$, and the forecast for Z_{n+1} is

$$\hat{\hat{Z}}_{n+1} = \bar{z} + \hat{\tilde{z}}_{n+1}.$$

(3) The forecasts for Z_{n+2}, …, $Z_{n+\ell}$ are just

$$\hat{\hat{Z}}_{n+k} = \bar{z}, \quad k > 1.$$

(4) 20 points Consider an MA(4) process of the form

$$Z_t \;=\; a_t \;+\; \theta_4 a_{t-4}.$$

Write this as an AR(∞) process of the form

$$Z_t \;=\; \sum_{k=1}^{\infty} \pi_k Z_{t-k} \;+\; a_t.$$

What is the range of θ_4 values for which the process is invertible?

<u>**Solutions:**</u> In operator notation

$$Z_t \;=\; (1+\theta_4 B^4) a_t.$$

By Box/Jenkins notation, it should probably be $-theta_4$, but that won't make much difference as long as we are consistent. Then computing the inverse in

$$a_t \;=\; (1+\theta_4 B^4)^{-1} Z_t,$$

we see from the geometric series

$$(1+v)^{-1} \;=\; \sum_{k=0}^{\infty} (-v)^k$$

that

$$\begin{aligned} a_t \;&=\; \sum_{k=0}^{\infty} (-\theta_4 B^4)^k Z_t \\ &=\; Z_t \;+\; \sum_{k=1}^{\infty} (-\theta_4 B^4)^k Z_t \\ &=\; Z_t \;-\; -\theta_4 Z_{t-4} + \theta_4^2 Z_{t-8} - \theta_4^3 Z_{t-12} + \ldots \end{aligned}$$

so the AR(∞) form has the coefficients

$$\pi_k \;=\; \begin{cases} -(-theta_4)^j & \text{if } k = 4j \text{ for some positive integer } j, \\ 0 & \text{otherwise.} \end{cases}$$

The invertibility condition would be based on the roots of

$$1 + \theta_4 v^4 \;=\; 0,$$

which are $\pm|\theta_4|^{-1/4}$ and $\pm i|\theta_4|^{-1/4}$. These are outside the unit circle provided $|\theta_4| < 1$.

(5) 20 points To the same data set, one statistician fits an MA(2) model of the form

$$Z_t \;=\; a_t - 1.0a_{t-1} + .09a_{t-2},$$

while another statistician fits an ARMA(1,1) model of the form

$$Z_t \;=\; -.1Z_{t-1} + a_t - .9a_{t-1}.$$

The diagnostics for the fit models do not show any signficant lack of fit for either, and the two statisticians spend hours arguing that their own models are the "right ones."

Show in fact that the two models are almost equal.

Solutions: We can write the MA(2) model as

$$Z_t \;=\; (1 - B + .09B^2)a_t \;=\; (1 - .9B)(1 - .1B)a_t,$$

and the ARMA(1,1) model as

$$(1 + .1B)Z_t \;=\; (1 - .9B)a_t.$$

The key to the claim that the two models are almost equal is that

$$(1 + .1B)^{-1} \;=\; 1 - .1B + .01B^2 - .001B^3 + .0001B^4 - \ldots \;\doteq\; 1 - .1B$$

and hence the ARMA(1,1) model is

$$Z_t \;=\; (1 + .1B)^{-1}(1 - .9B)a_t \;\doteq\; (1 - .1B)(1 - .9B)a_t$$

which is the MA(2) model.

(6) 44 points A statician analyzes a series z_t with 200 observations in Splus. Below is the Splus code:

```
> par(mfrow=c(3,1))
> tsplot(z)
> acfz_acf(z)
> pacfz_acf(z,type="partial")
> par(mfrow=c(1,1))
> fit.ar1_arima.mle(z,model=list(ar=0))
> diag.ar1_arima.diag(fit.ar1)
> fit.ar4_arima.mle(z,model=list(ar=c(0,0,0,0)))
> diag.ar4_arima.diag(fit.ar4)
> fit.ma4_arima.mle(z,model=list(ma=c(0,0,0,0)))
> diag.ma4_arima.diag(fit.ma4)
```

Figure 3 shows the time series plot, sample ACF, and sample PACF. Figure 4 shows the output from the diagnostics of the AR(1) fit. Figure 5 shows the output from diagnostics fo the AR(4) fit. Figure 6 shows the output from the diagnostics of the MA(4) fit.

Based on this information, which of the three fits do you believe is the best? Do you believe any of the three fits is adequate? Justify your answers.

Solutions: The ACF in Figure 3 suggests an MA(4) model and the PACF in the bottom of Figure 3 suggests either an AR(1) model (if we ignore the slightly significant PACF value at lag 5) or an AR(5) model. Of course, a mixed model may also be appropriate, but that is not considered here. The AR(1) model is treated first in Figure 4. The residual plot (top panel in Figure 4) seems OK but the residual ACF (middle panel in Figure 4) shows a significant value at lag 4, and the Portmanteau test (bottom panel in Figure 4) shows signficant values for all lags ≥ 4. Based on the latter two observations, we cannot regard the AR(1) model as adequate.

Regarding the diagnostics for the AR(4) model in Figure 5, the residual time plot doesn't show much (looks pretty much the same as in the previous case) and the residual ACF shows slightly significant values at lags 4 and 5, but the Portmanteau test gives significant values for all lags (≥ 4), so this model seems inadequate as well. An AR(5) model might have worked to pass the Portmanteau test, but we aren't considering it.

Moving on to the MA(4) model as diagnosed in Figure 6, there appears a slightly significant value in the residual ACF at lag 11 but the Portmanteau test statistic is insignificant for all lags depicted (4 through 13), so we conclude that there is not strong evidence for the invalidity of this model whereas there is strong evidence for the invalidity of the other two.

The MA(4) model is clearly the best, given the available information.

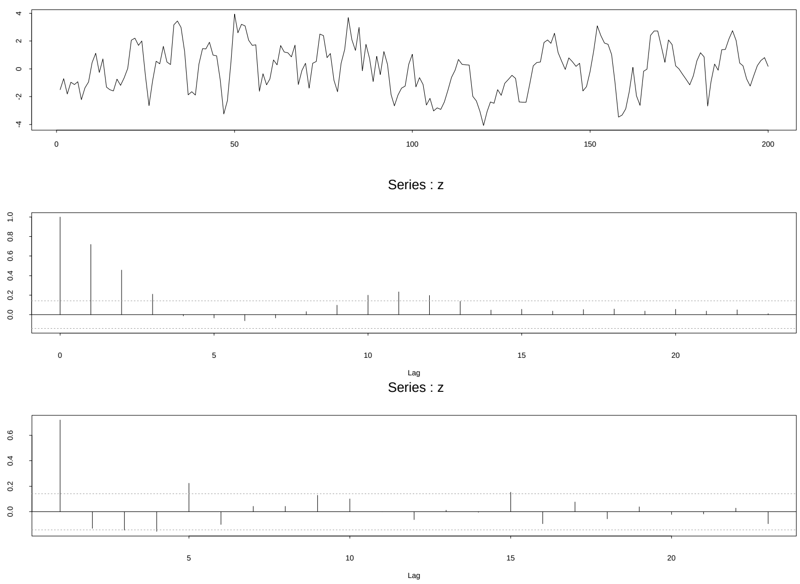

Figure 3: Time Series plot, sample ACF, and sample PACF in Problem 6.

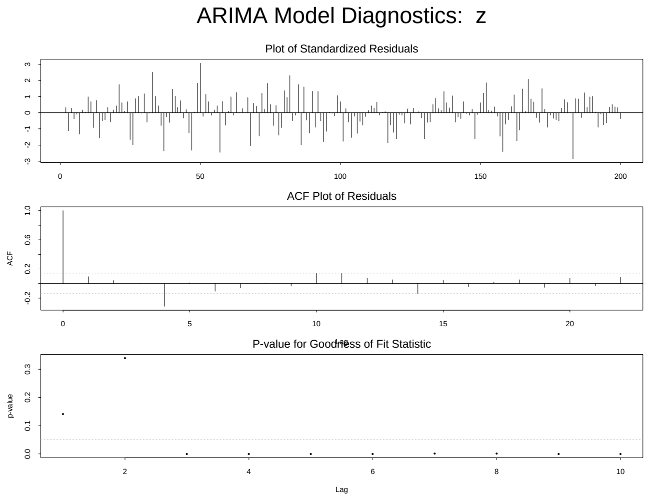

Figure 4: Diagnostic plots for AR(1) model fit to series in Problem 6.

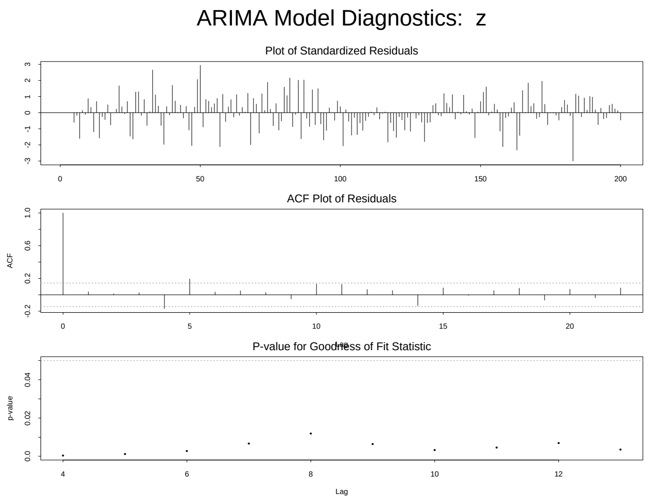

Figure 5: Diagnostic plots for AR(4) model fit to series in Problem 6.

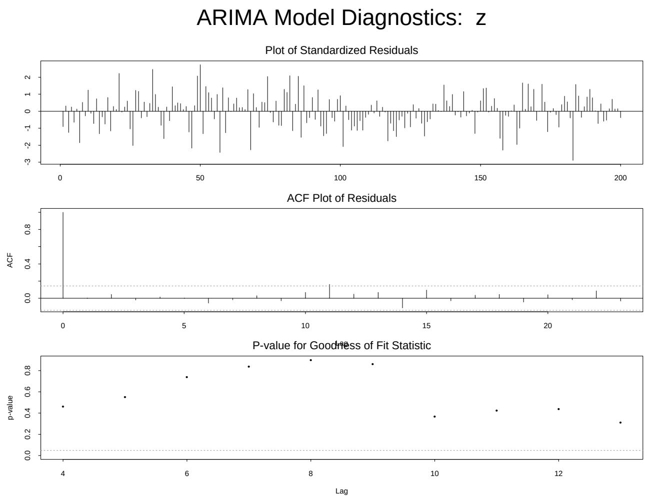

Figure 6: Diagnostic plots for MA(4) model fit to series in Problem 6.