

# Computational Finance – A Statistical Approach

J.A. Dobelman

Math-Science Scholars Program

Math Camp

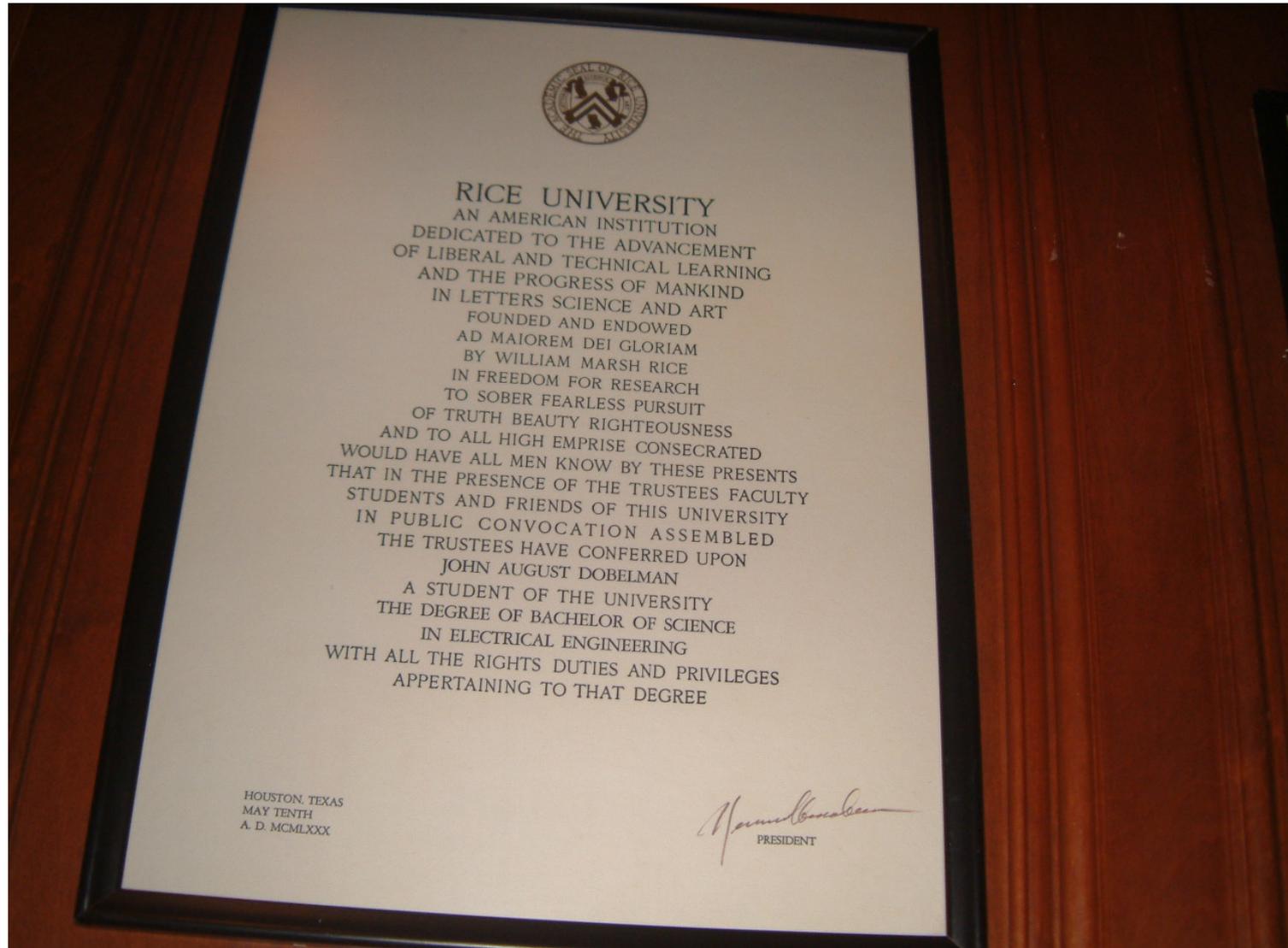
June 12, 2017

**RICHARD TAPIA CENTER**  
FOR EXCELLENCE & EQUITY

- How did we get here?
- What does *Math* have to do with real life?
- What does *Finance* have to do with real life?
- What is the deal with computational finance?
- What is a statistical approach to computational finance?

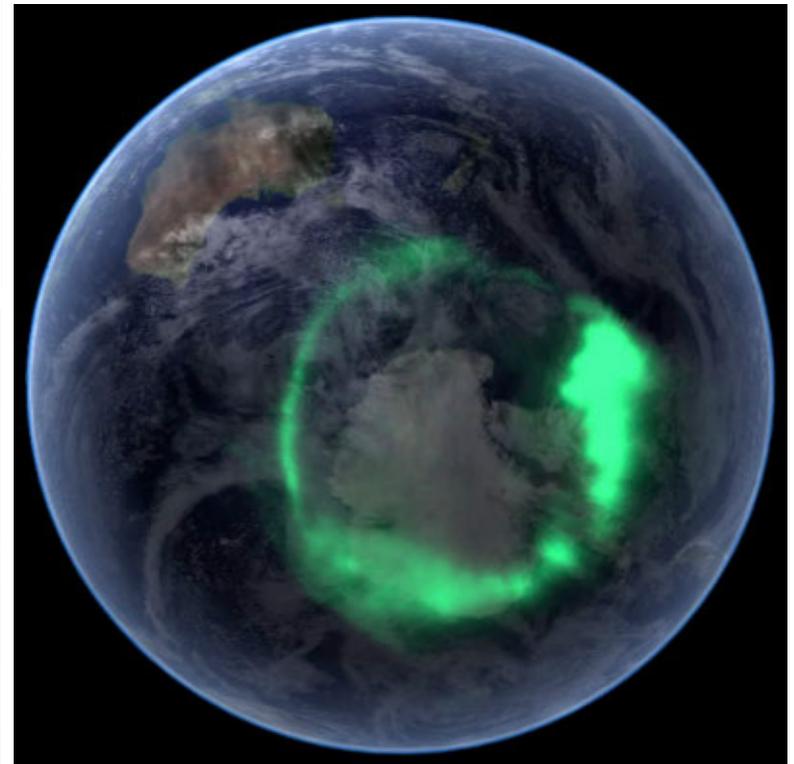
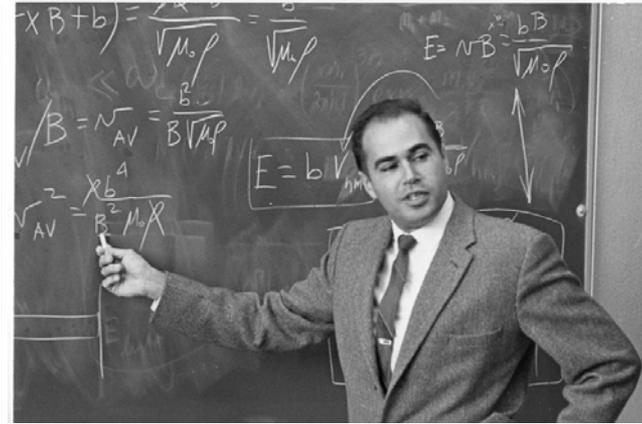
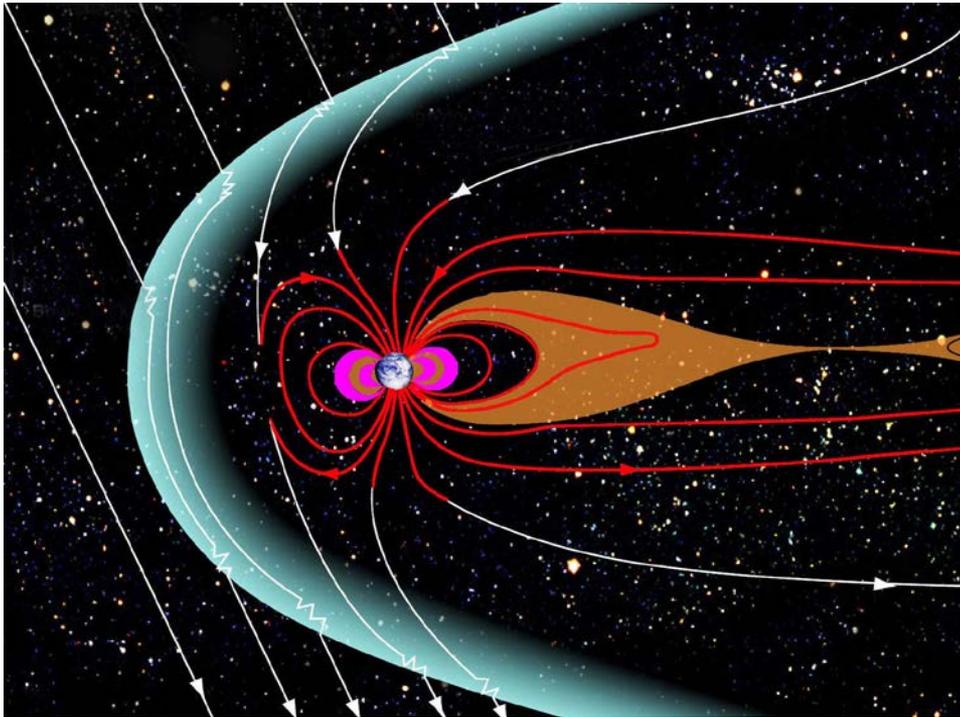
$$e^{-j\pi} + 1 = 0$$

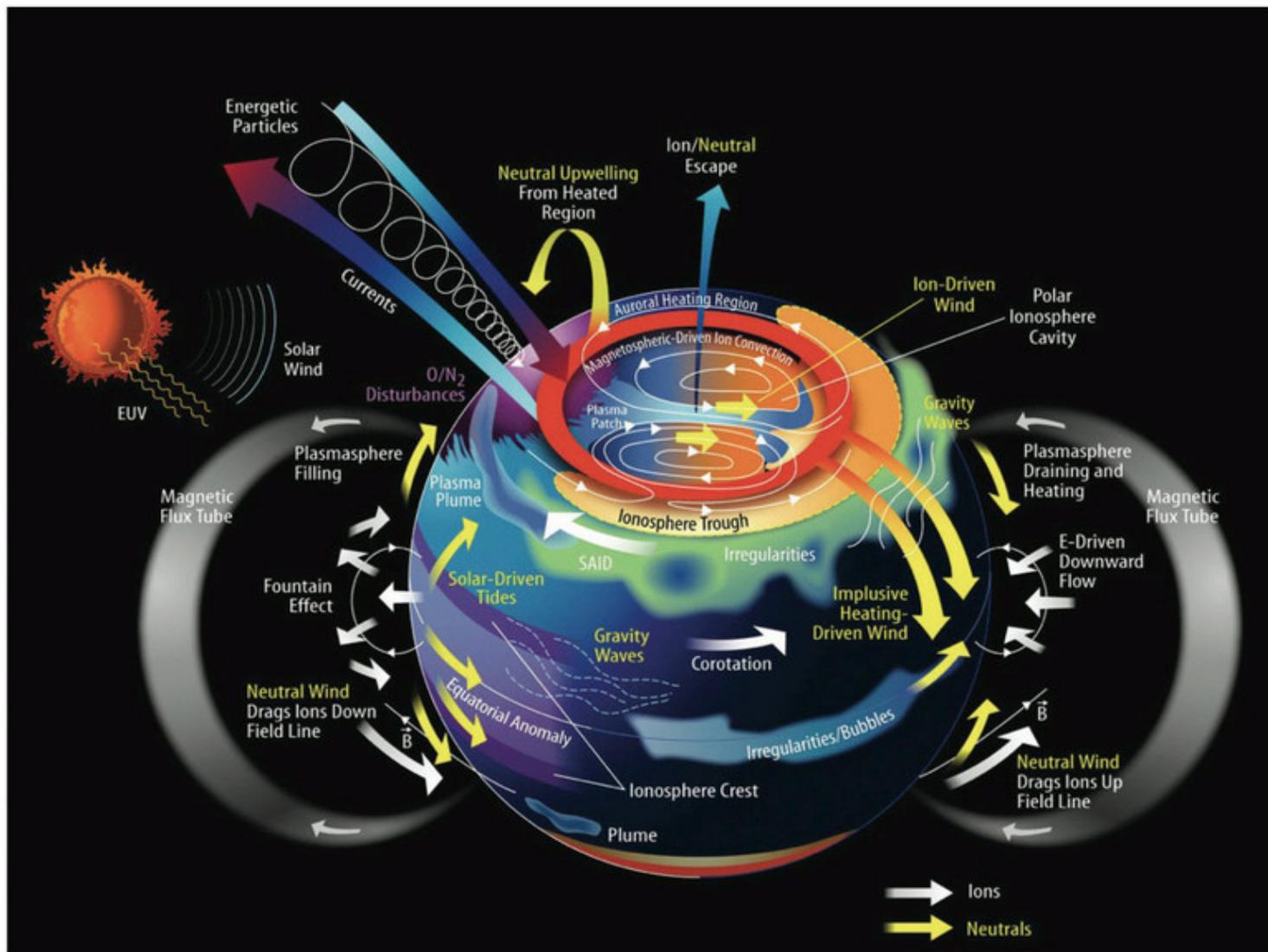




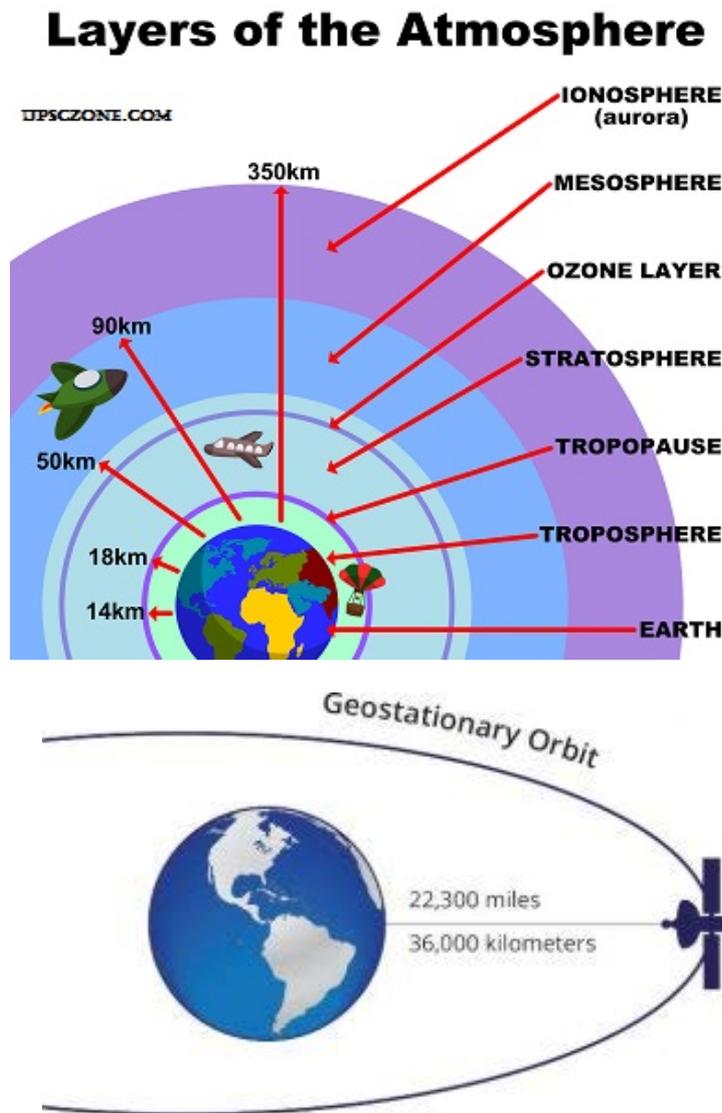


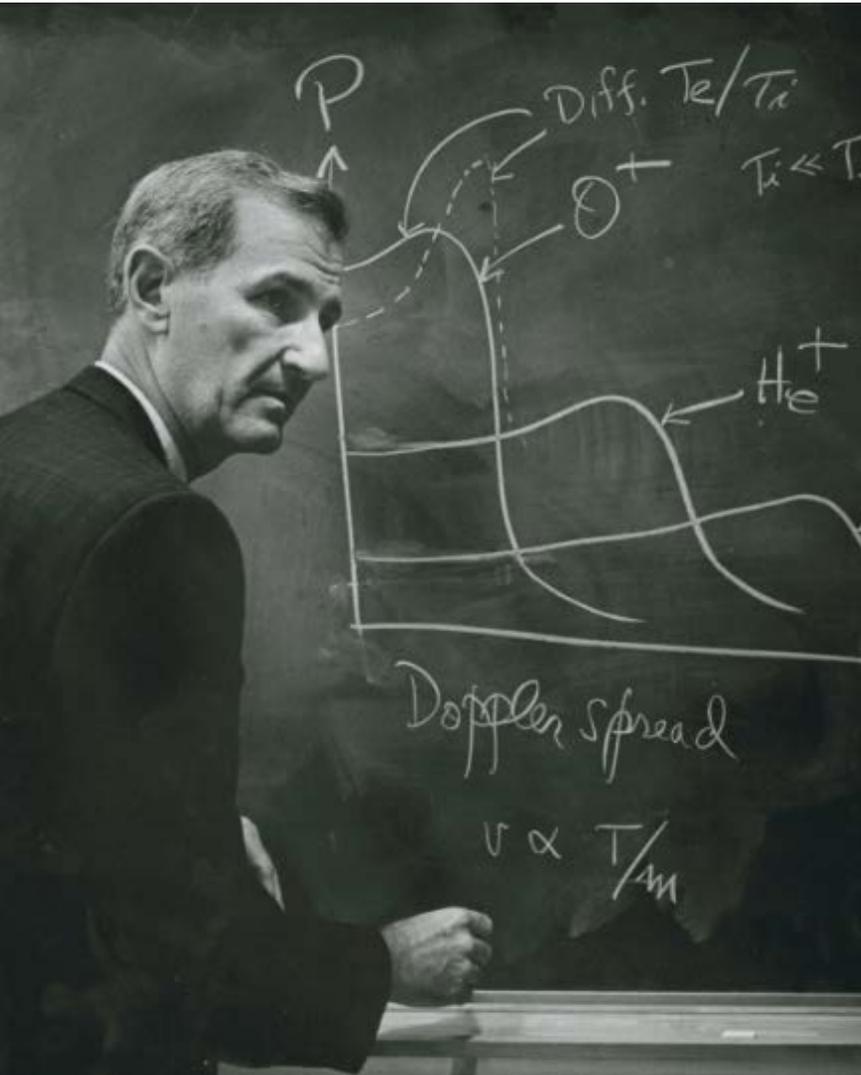
First post-college job: Bindery Operator





- Enroute aircraft (5 mi)
- Troposphere 20 km (12 mi)
- Ionosphere 50-400 km (30-240 miles)
- Low Earth Orbit (100mi/160 km)
- Geosynchronous orbit (22,236 mi)





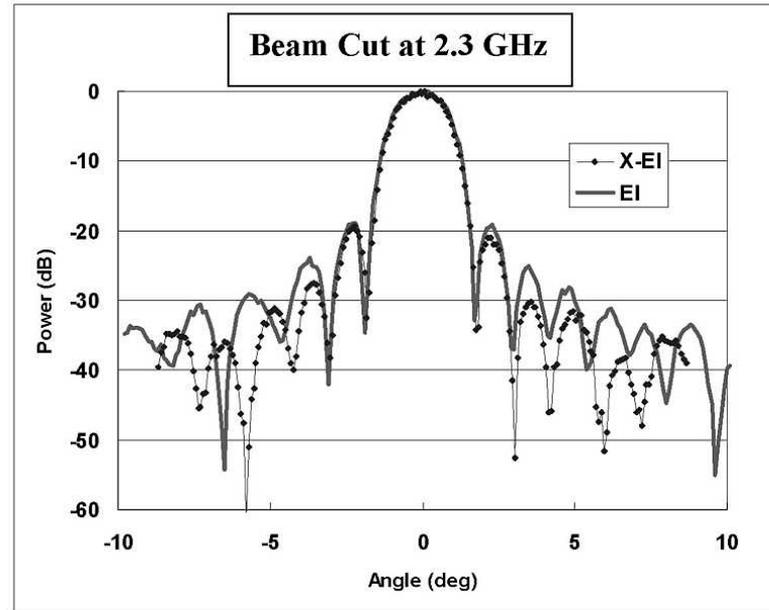
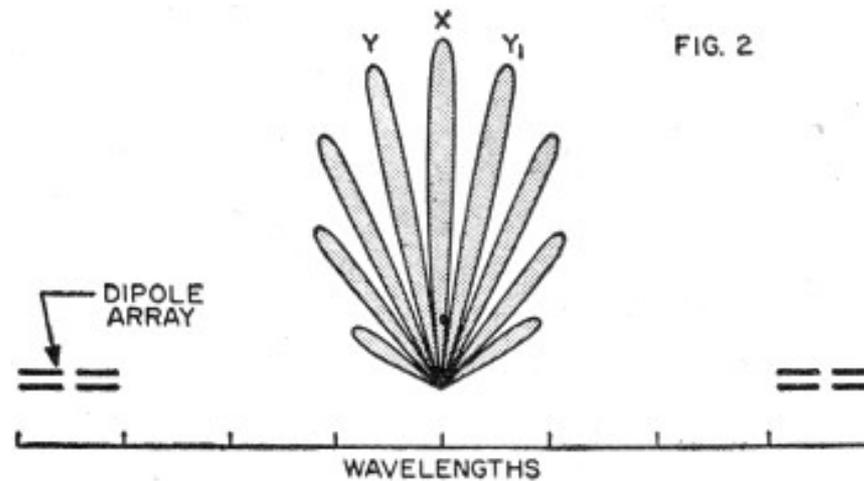


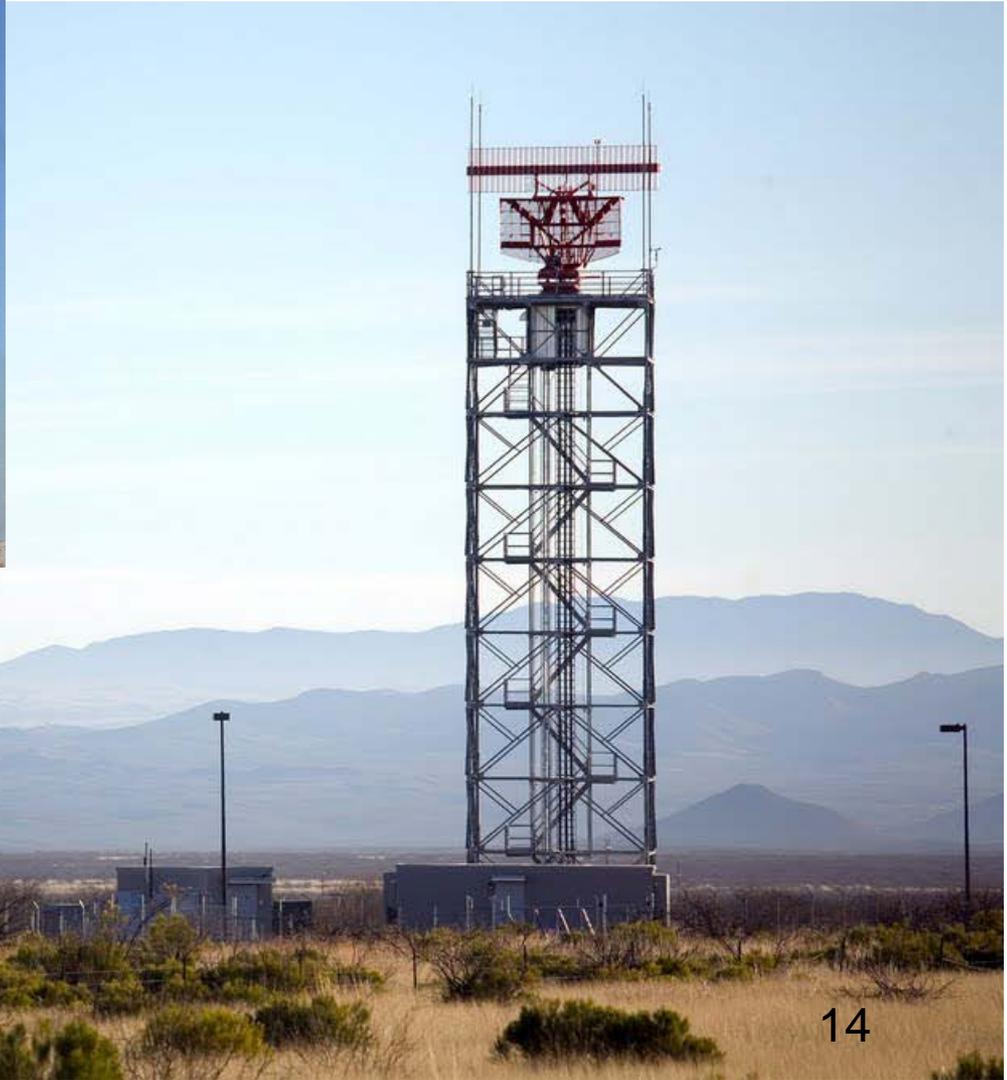
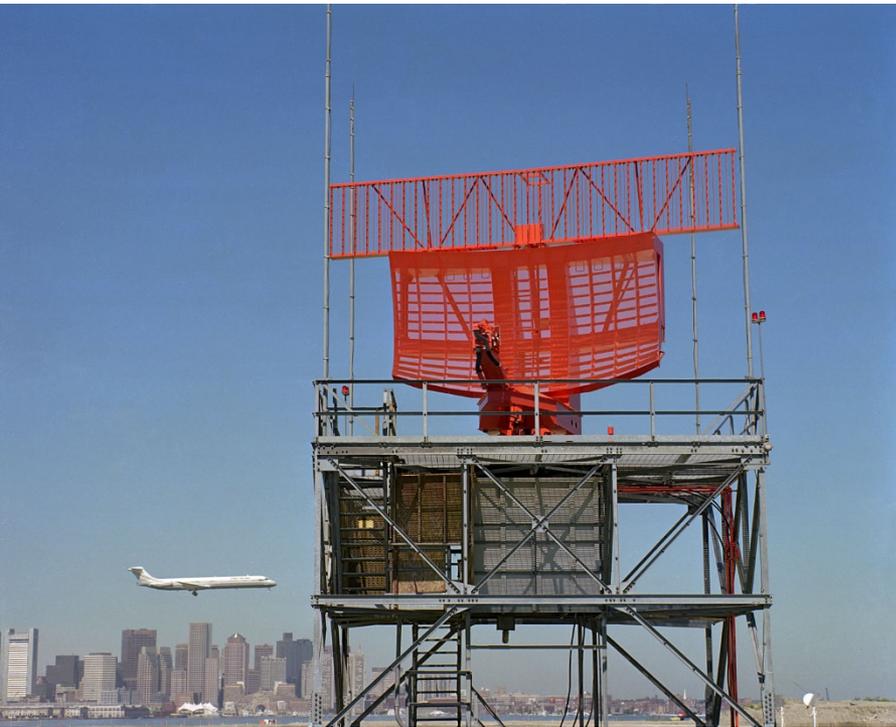
Figure 1: Two cuts through the primary beam pattern of one of the ATA dishes.



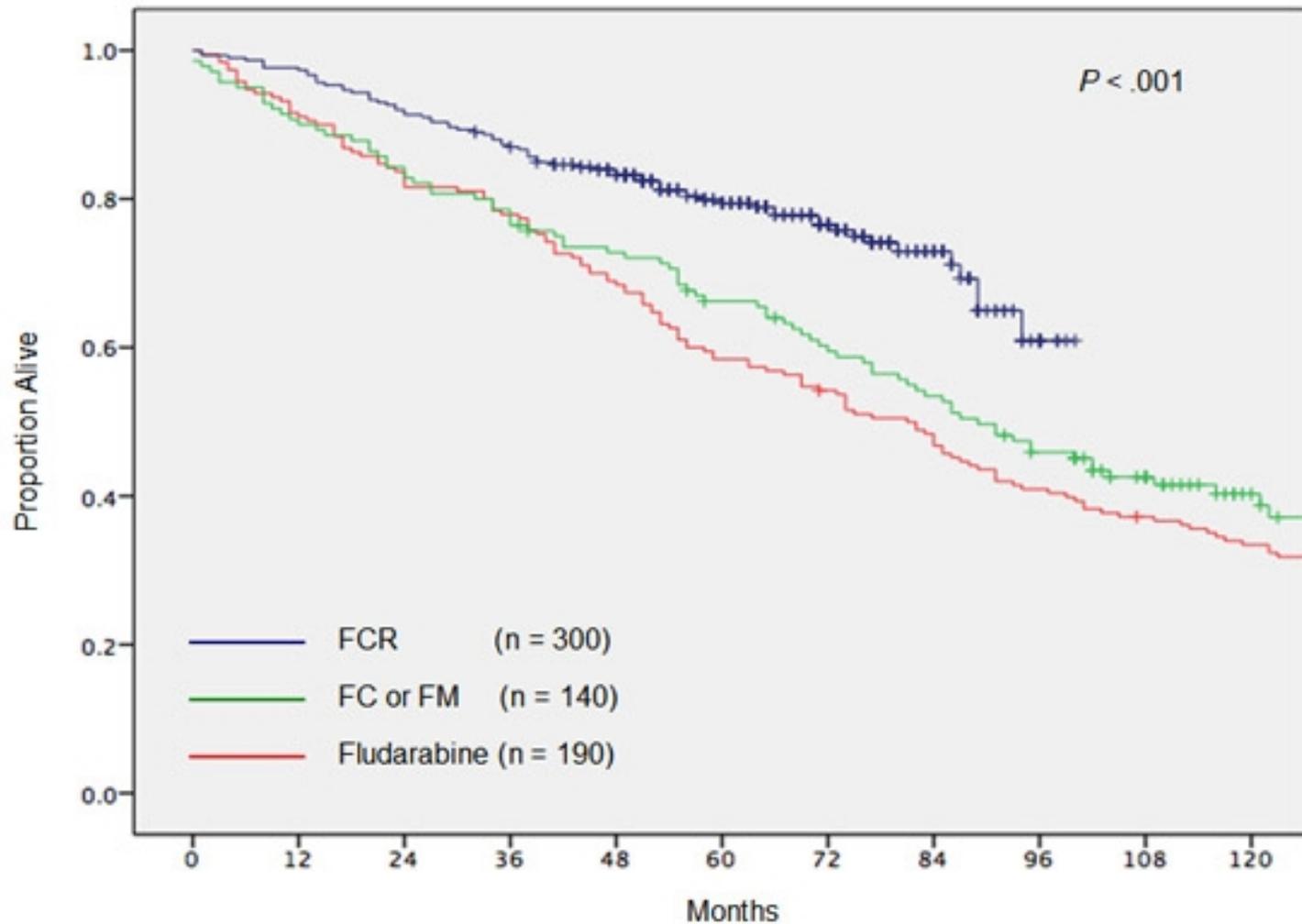


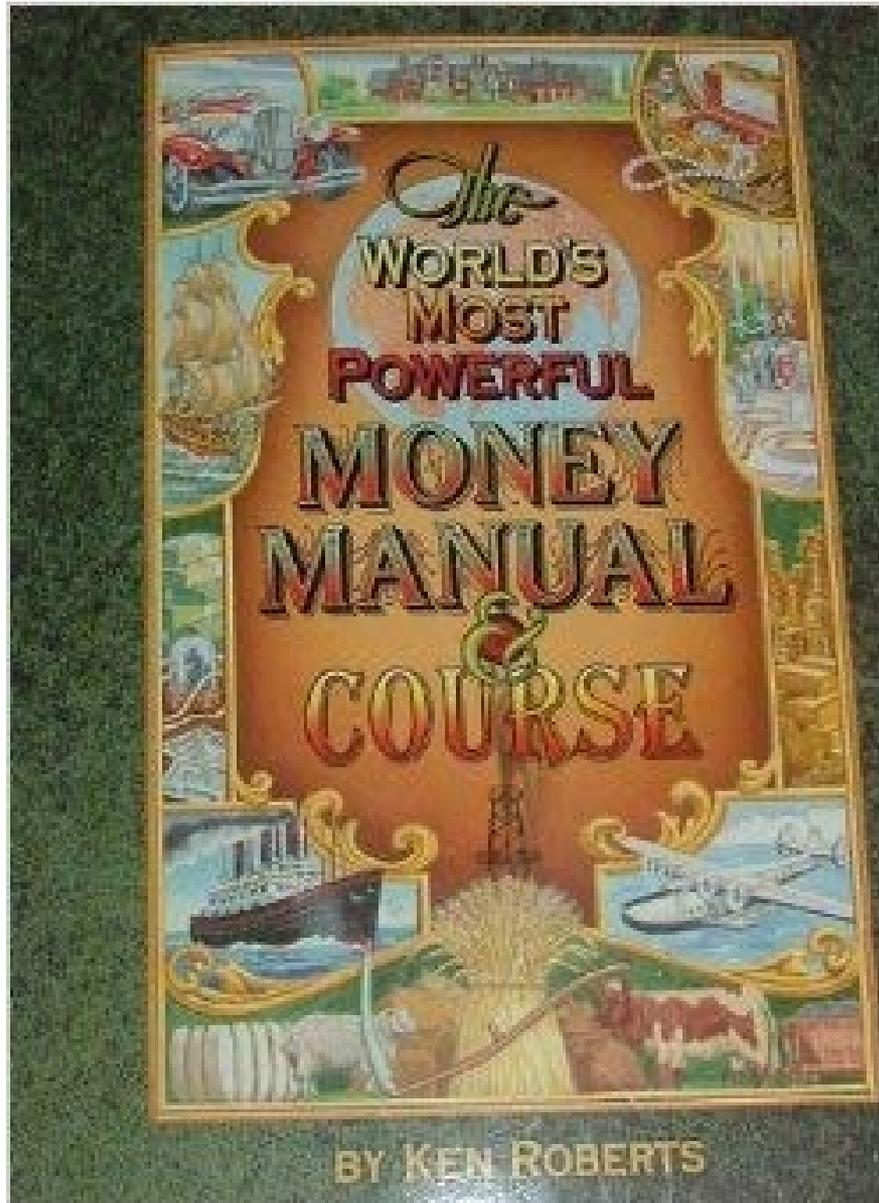












**\$500 a week!**



CC: 10T

CT: 50k#

JO: 15k#

KC: 37.5k# (250 bags)

18



- More than puzzles and unsolved ?'s

Algebra

Algebraic geometry

Analysis

Combinatorics

Discrete geometry

Euclidean geometry

Dynamical systems

Graph theory

Group theory

Model theory

Number theory

Additive number theory

Algebraic number theory

Combinatorial number theory

Prime numbers

Partial differential equations

Ramsey theory

Set theory

- 
- The background of the slide is a photograph of numerous stacks of US dollar bills, arranged in a grid-like pattern. Each stack is bound with a white paper band that has "\$10,000" printed on it in black ink. The bills are slightly fanned out, showing the top of each bill. The lighting is bright, highlighting the texture of the paper and the colors of the bills.
- Millennium Prize Problems (Clay Mathematics Institute in 2000)
    - P versus NP (CS)
    - Hodge conjecture (analytic geometry)
    - Riemann hypothesis (1859; zeta fn. zeros)
    - Yang–Mills existence and mass gap (physics)
    - Navier–Stokes existence and smoothness (fluid mechanics and turbulence building blocks)
    - Birch and Swinnerton-Dyer conjecture (number theory)



- “Quants” turned finance world upside down in 1973
- 30 years later “Data Mining” emerges
- Mathematics and computer science
- Modeling YOU as a customer, a specimen and a target
- Behavioral prediction
  - Revenue management and optimization
  - Surveillance
- AI/Robotics

- Science and Engineering
  - Mathematical biology & bioinformatics
    - CURE for cancer
    - GROW new organs
    - FOREVER YOUNG
  - Nanotechnology

- Engineering
  - Bioengineering
  - Chemical and Biomolecular Engineering
  - Civil and Environmental Engineering
  - Computer Science
  - Electrical and Computer Engineering
  - Materials Science and NanoEngineering
  - Mechanical Engineering
  - Computational and Applied Math
  - Statistics

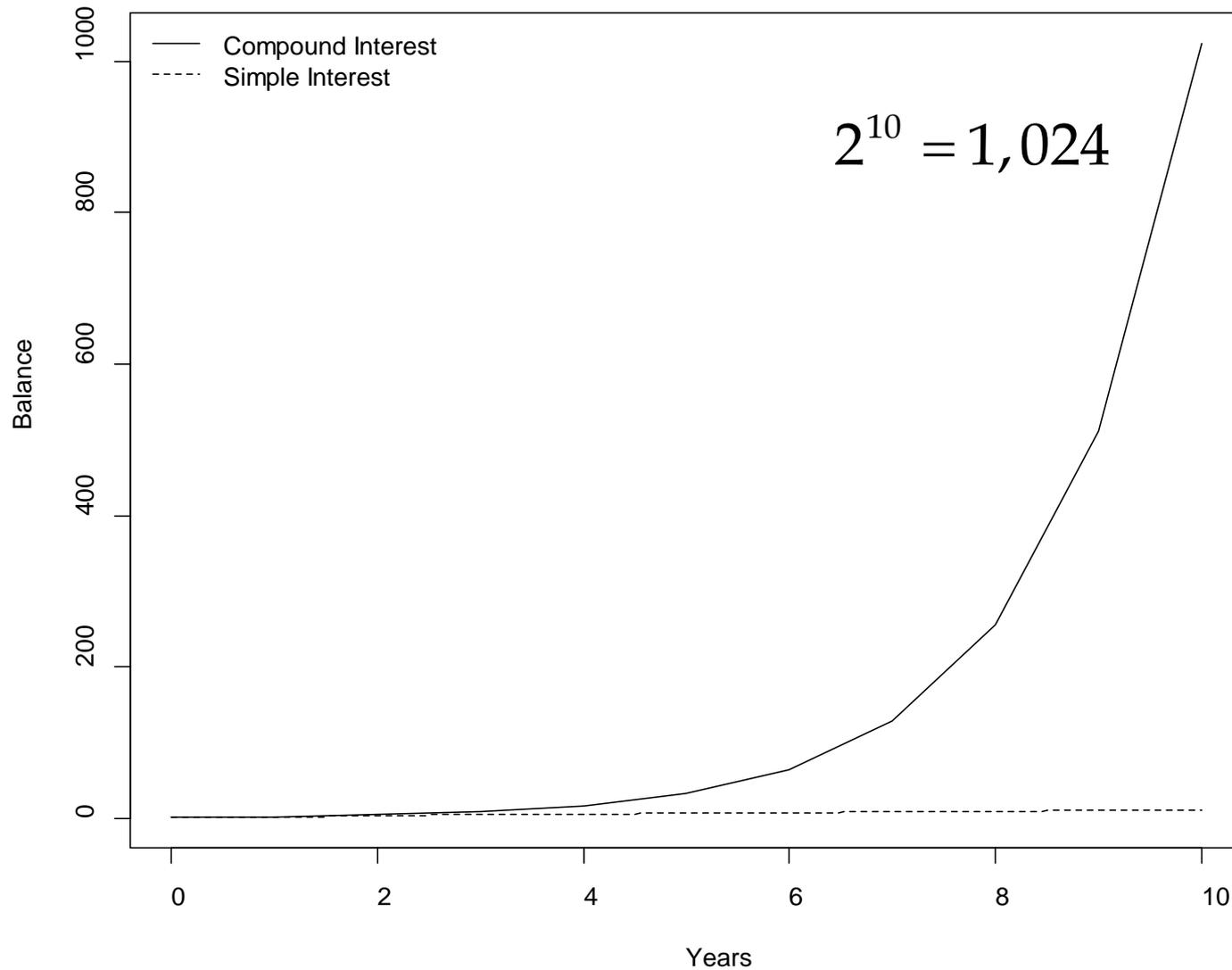
- In previous TMSS lectures we discussed making money
- The magic of compounding and regular contributions was discussed
  - Suppose one doubles their money every year
  - This is a compounding rate of 100% per year

$$(1 + r)^n, r = 1.00 \rightarrow (2)^n$$

$$R_n = 2^n, n = 0, 1, 2, \dots$$

$$2^{n=3} = 2^1 \cdot 2^1 \cdot 2^1 = 2^3 = 8$$

## Compound vs. Simple Interest Rate = 100%



- Estimate of all the money in the world
  - Money, bills & coins: \$5 trillion
  - Add checking accounts: \$25 trillion
  - Add savings and CD's < \$100,000: \$60T
    - \$60T = \$60,000B =  $6 \times 10^{13}$ .
- Before long....

$$2^n = 6E13$$

$$n = \log_2(6E13) = \frac{\log(6E13)}{\log(2)} = 45.77 \text{ years}$$

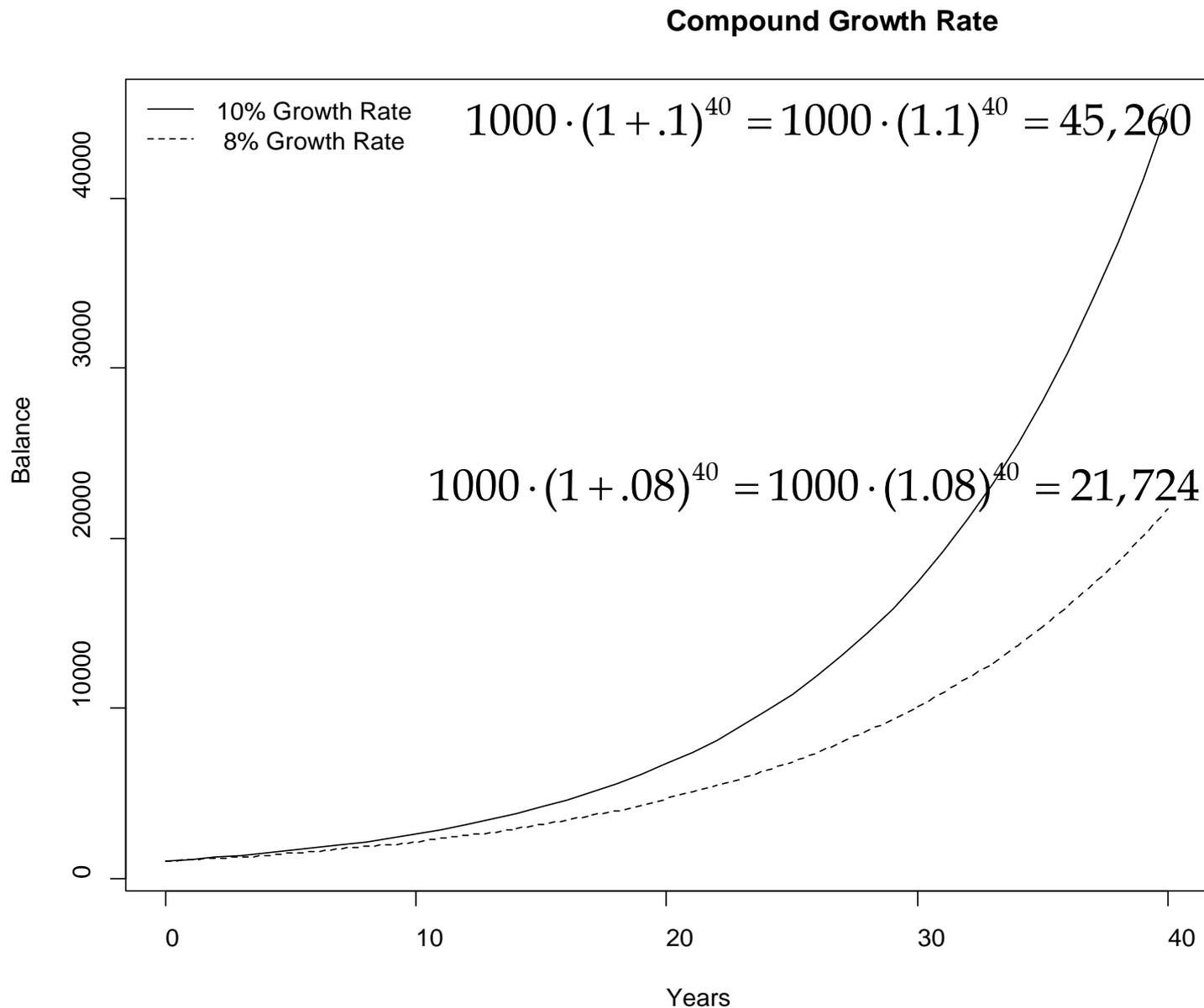
- If your money tripled each year

$$R = (1 + 2.0)^n = (3)^n$$

$$3^n = 6\text{E}13$$

$$n = \log_3(6\text{E}13) = \frac{\log(6\text{E}13)}{\log(3)} = 28.9 \text{ years}$$

- Lets be more realistic
  - $r = 8\%$ , or  $10\%$  per year
  - Stock market with dividends reinvested
  - Start with \$1,000, no more additions for 40 years



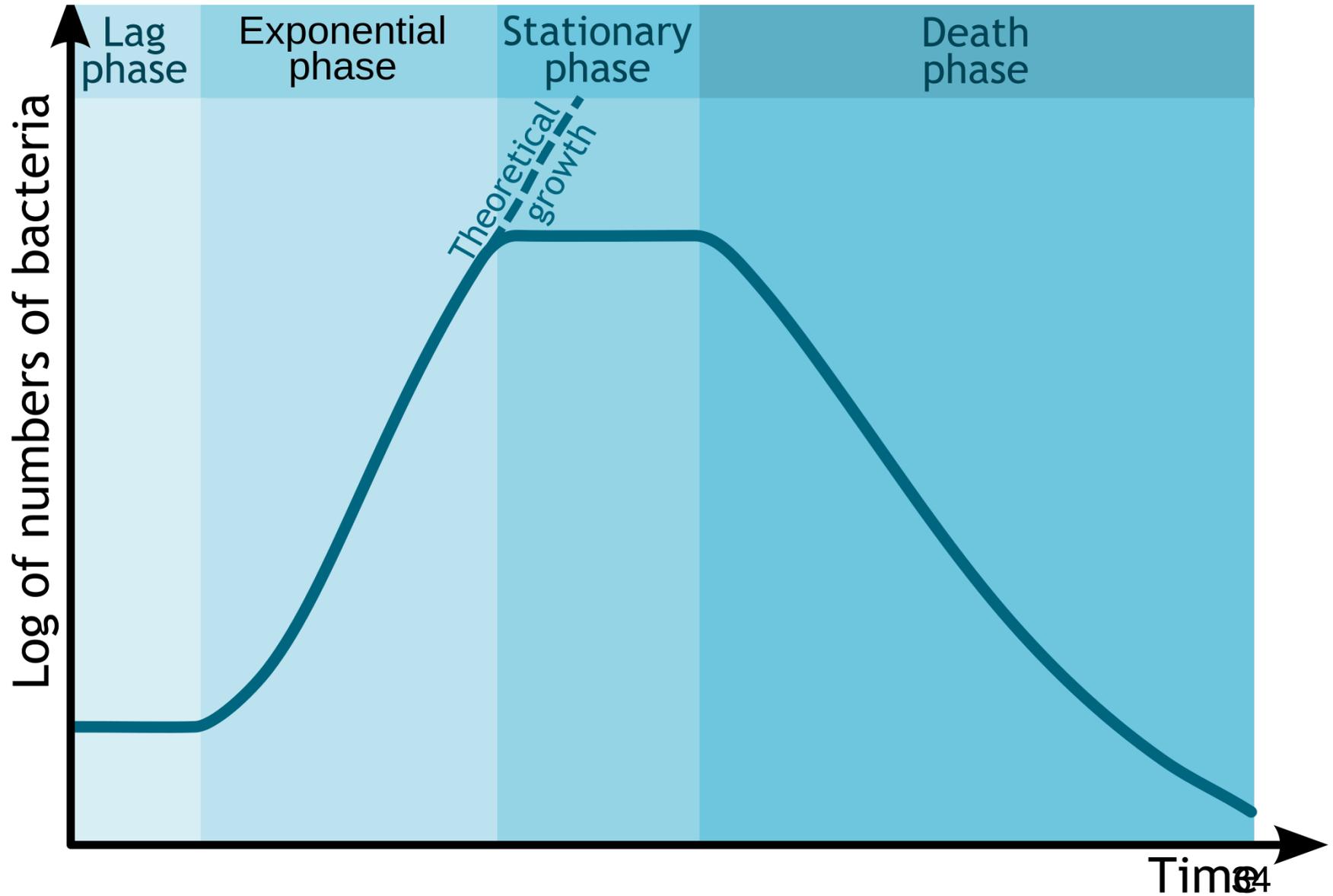
- Compounding miraculous
  - “Gentlemen, if the man who invented compound interest had of secured a patent on his idea he would have had without any doubt the greatest invention the world has ever produced.” - Security Investment Co., 1916.
  - Oftentimes attributed to Einstein
- Compounding unsustainable
  - Even at 8%, after 422 years all the money in the world would be gone

- “And there came a certain poor widow, and she threw in two mites, which make a farthing” Mk 12.42
- Worth one-half of a quadrans or 1/128 of a denarius, or about six minutes of an average daily wage
  - $\$15 \times 8\text{hrs} = \$120/\text{day}$
  - $60 \times 8 = 480 \text{ min. per day}$
  - $6/480 = .0125 = 1.25\%$
  - $\$120 \times .0125 = \$1.50$



- Principal: \$1.50 (in today's money)
- Interest: 6% per annum (typical)
- Yeshua to Muhammed: \$13.9 Qn
- Time: 30 CE – 2015 CE (1,986 years)
- Taxrate: 0% 30%  
Value:  $2.713 \times 10^{50}$   $4.58 \times 10^{35}$
- $4.65 \times 10^{10}$  years at light speed to get to edge of the observable universe

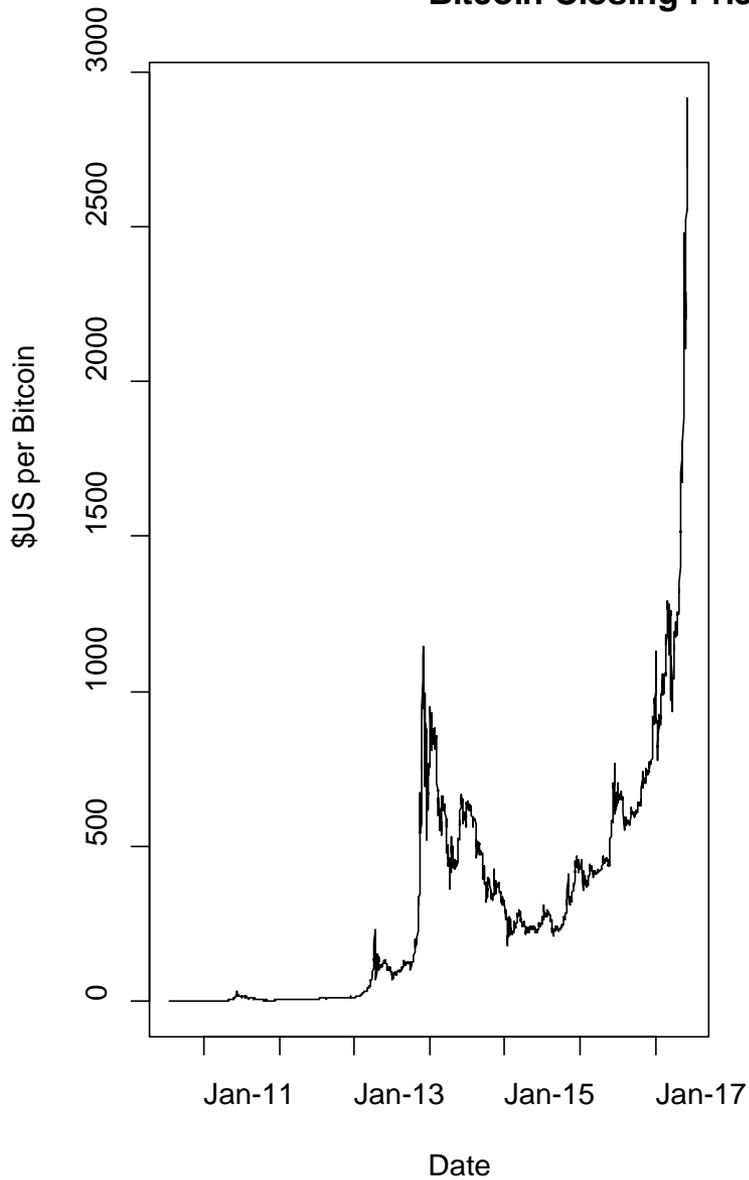




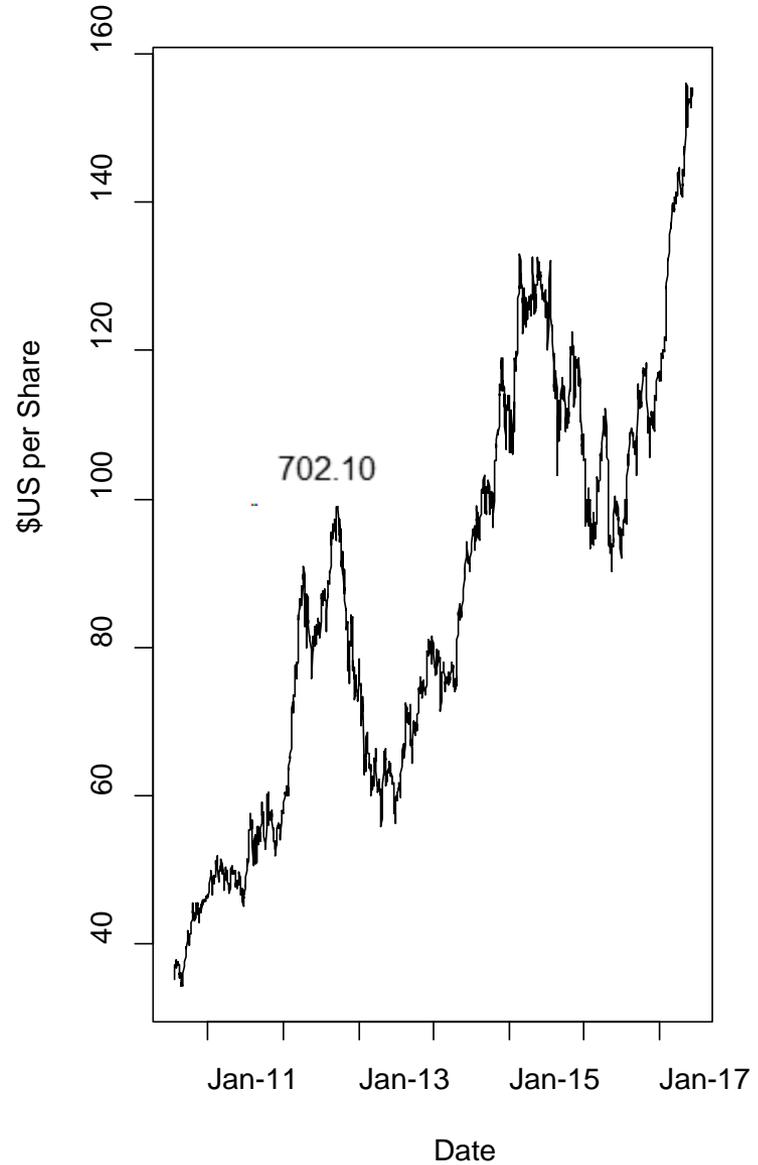
- Definition:
  - 1 plural : resources (as money) available to a government, person, group, or business
  - 2 : the obtaining or providing of funds or capital
  - 3 : the system that includes the circulation of money, the providing of banks and credit, and the making of investments

- What is Finance?
  - Personal (your money)
  - Academic (no money)
  - Professional (other peoples money)
- Capital Markets
  - Funding for chop shop
  - How to get financing for SNAP
- Asset pricing
  - AAPL
  - Bitcoin

### Bitcoin Closing Prices



### AAPL Price (Adjusted)



$$7 / 18 / 10 - 6 / 8 / 17 = 2517 \text{ days} \rightarrow \text{Years} = \frac{2517}{365.25} = 6.89$$

- AAPL (CAGR)

$$R_T = \frac{155.37}{35.08} = 4.428$$

$$(1 + r^*)^n = R_T$$

$$r^* = \sqrt[n]{R_T} - 1 = \sqrt[6.89]{4.428} - 1 \\ = 0.24 = 24\%$$

- AAPL (TV)

\$100 → \$443

(3/4 iPhone)

- Bitcoin (CAGR)

$$R_T = \frac{2763}{.08} = 32,212$$

$$(1 + r^*)^n = R_T$$

$$r^* = \sqrt[n]{R_T} - 1 = \sqrt[6.89]{32212} - 1 \\ = 3.50 = 350\%$$

- Bitcoin (TV)

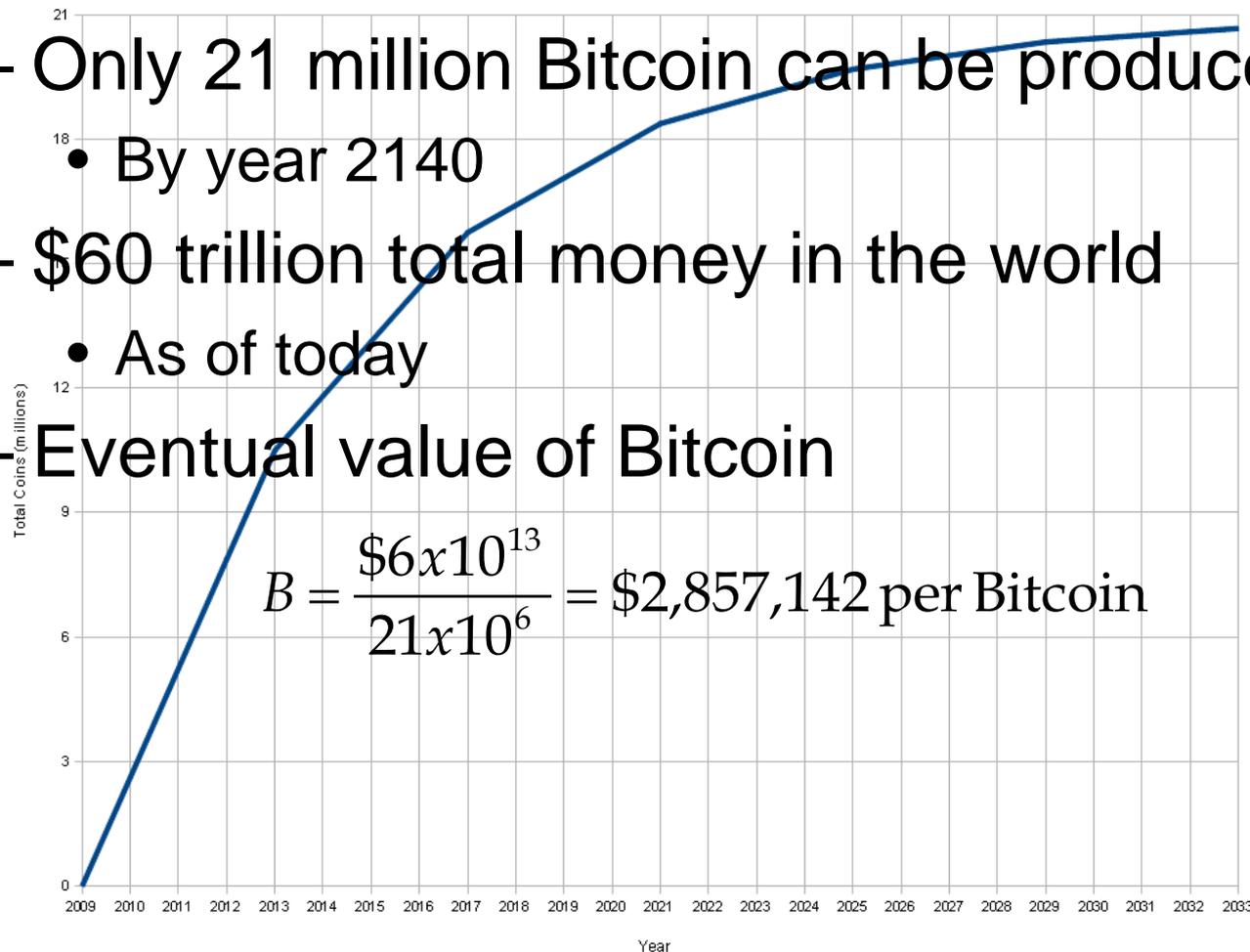
\$100 → \$3,221,200

(5,369 iPhones)

- **NOTE:**

Total Bitcoins over time

- Only 21 million Bitcoin can be produced
  - By year 2140
- \$60 trillion total money in the world
  - As of today
- Eventual value of Bitcoin



$$B = \frac{\$6 \times 10^{13}}{21 \times 10^6} = \$2,857,142 \text{ per Bitcoin}$$

- A doctoral degree in finance **qualifies** you to pursue an academic career as a researcher and educator. An academic career provides not only tremendous intellectual **freedom**, but makes it possible to work with **bright people** throughout your life. You can make a difference through the scholarly **research** you create, and the lives you shape through your **teaching**.

- Not to mention:
  - The starting salary for finance PhD graduates' often ranges from \$180K to \$210K, **depending on placement**, which is largely determined by performance in the doctoral program.
  - The job market for finance PhDs has been strong, and in recent years **more positions** have been available than candidates pursuing jobs.
  - The position of a university professor is frequently ranked as one of the **most desirable** jobs in the world

Stanford Graduate School of Business

NYU Stern Business School

Univ. Pennsylvania Wharton School

MIT Sloan School

University of Texas - McCombs Business School

Columbia University - Columbia Business School

Northwestern University, Kellogg School of Management

Rutgers Business School

University of California Berkeley, Haas Business School

University of Colorado - Leeds School of Business

Emory University - Goizueta Business School

University of Chicago - Booth School of Business

“XXX is a good place for learning what it is to be an academic – a member of the finance and economics community – and what that entails.”

- Accounting
- Business & Public Policy
- **Finance**
- Management of Organizations
- Marketing
- Real Estate

- Structure of financial markets
- Formation and behavior of financial asset prices
- Banking and monetary systems
- Corporate control and capital structure
- Saving and capital formation
- International financial markets

- Formation and behavior of financial asset prices
  - Asset pricing models
  - Stochastic modeling
    - Real and Functional Analysis
    - Theory of Probability
    - Stochastic Processes
    - Dynamic Optimization
  - Time-series econometrics
    - Time Series Analysis
    - Topics in Financial Econometrics

## The Berkeley-Haas PhD Program Finance

### Requirements Years 1 and 2:

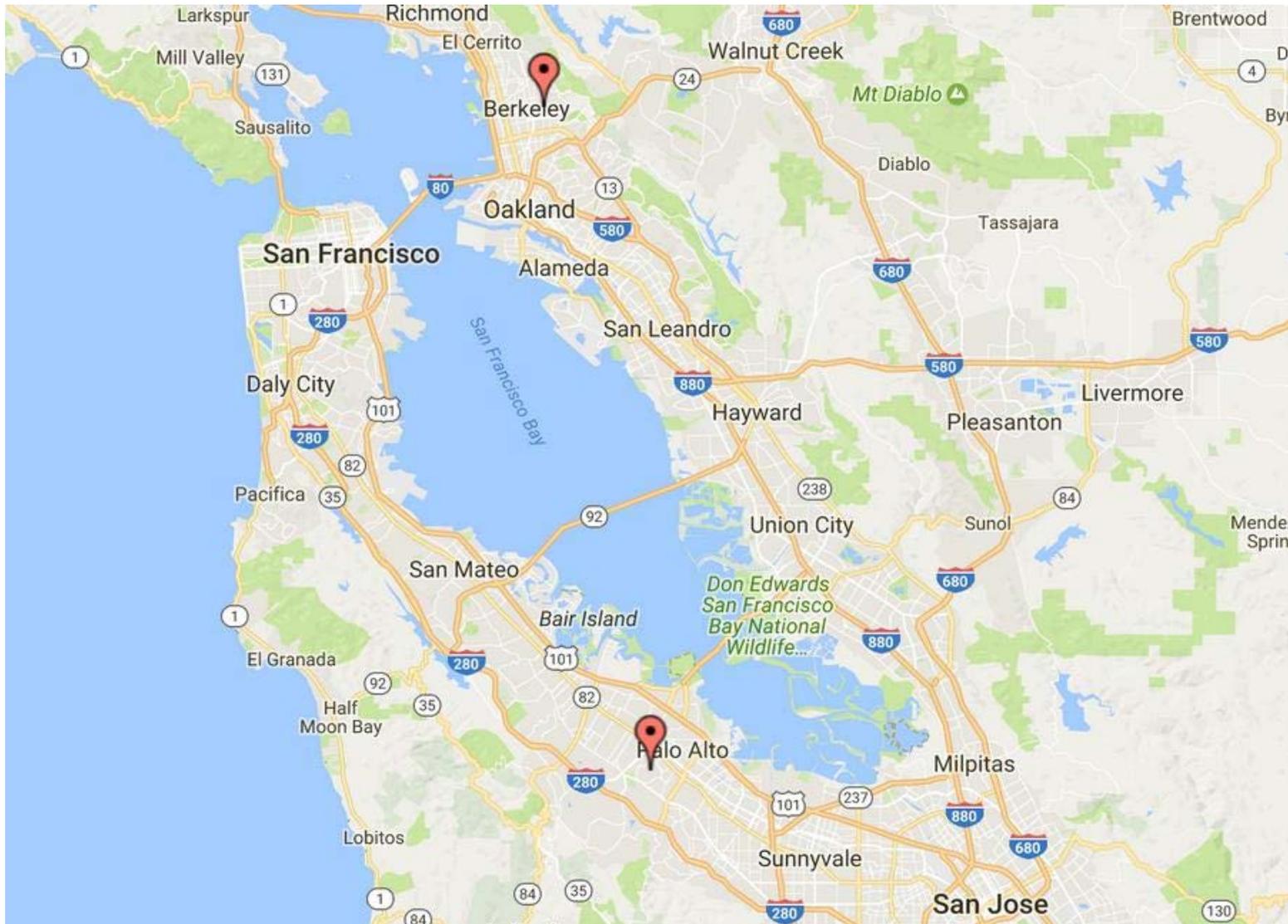
**Core Classes:** Required and used to determine overall GPA; need 3.3 cumulative GPA in these classes.

<b>1<sup>st</sup> Semester - Fall</b>	<b>2<sup>nd</sup> Semester - Spring</b>	<b>3<sup>rd</sup> Semester - Fall</b>	<b>4<sup>th</sup> Semester - Spring</b>
Econ 201A	Econ 201B	PHDBA 239C Emp Fin	PHDBA 239DA ustructure
Econ 240A	Econ 240B	PHDBA 239S Seminar	PHDBA 239DB Corp fin
PHDBA 239A DTAP	PHDBA 375 Teaching		PHDBA 239S
	PHDBA 239B CTAP		

The table only outlines the required courses listed below. Students are expected to meet with the Finance Field Advisor to find additional courses or enroll in PHDBA 602 starting in their 3<sup>rd</sup> semester.

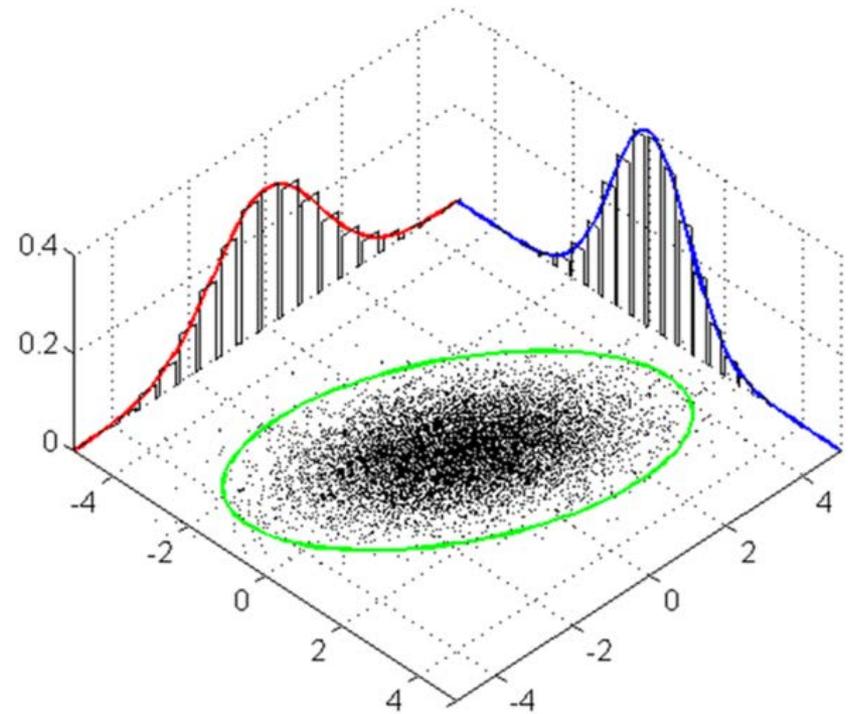
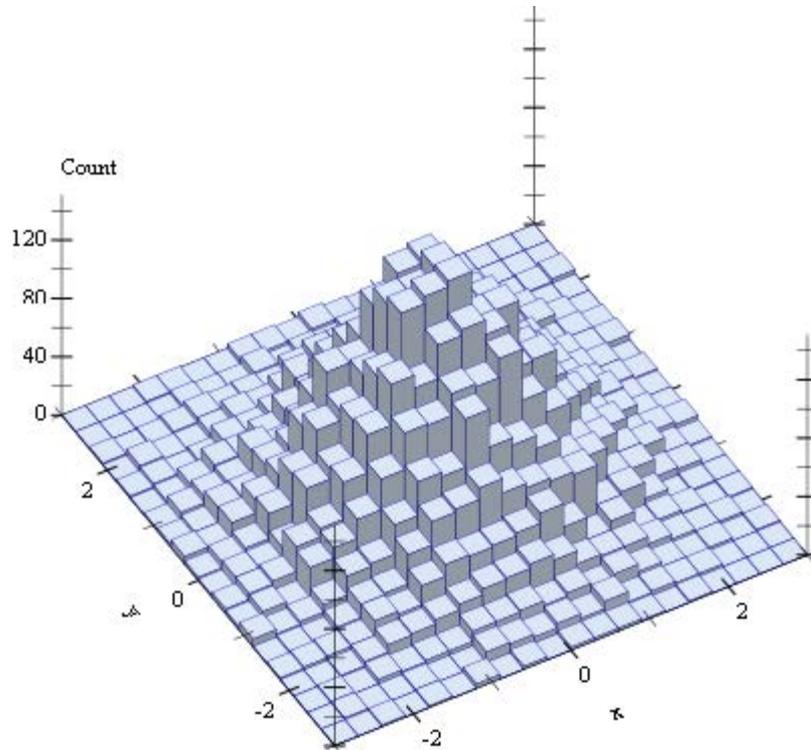
- Econ 204 Mathematical Tools for Economics, Prerequisite for Econ 201A; taught in the summer
- Econ 201A Economic Theory
- Econ 201B Economic Theory
- Econ 240A Econometrics
- Econ 240B Econometrics
- PHDBA 375 Teaching Business (required to work as a GSI)

# RICE UNIVERSITY



## PhD Curriculum

Course		Professor	Hours
<b>First Year</b>			
<b>Fall</b>			
ECON 7610	Math for Economists - Minor		3
ECON 7700	Price Theory I - Minor		3
FIN 7520	Seminar in Fin Research Methods - Research		3
<b>Spring</b>			
FIN 7550	Theory of Finance		3
ECON 7630	Econometric Methods I - Minor		3
<b>Second Year</b>			
<b>Fall</b>			
FIN 7750	Seminar in Corporate Fin		3
FIN 7855	Seminar in Options and Futures		3
ECON 7631	Econometric Methods II - Research		3
<b>Spring</b>			
FIN 7850	Seminar in Investments		3
ECON 7720	Price Theory II - Minor		3
ECON 7632	Econometric Theory III - Research		3
<b>Third Year</b>			
<b>Fall</b>			
FIN 7650	Seminar in Fin Markets and Inst.		3
ECON 7633	Econometric Theory IV - Research		3
<b>Any Time</b>			
MGT 7xxx	Breadth of Study		3
MKT 7xxx	Breadth of Study		3
ACCT 7xxx	Breadth of Study		3
Electives	7000 level courses in math or Experimental statistics		6
<b>Total</b>			<b>54</b>



**Lagrangian Probability Distribution Function (PDF)**

This can be transformed to the Stratonovich representation, in terms of the Feynman Lagrangian  $L$  possessing a covariant variational principle,

$$P[M_T | M_{t_0}] dM(t) =$$

$$\int \cdots \int \underline{DM} \exp \left( - \min \int_{t_0}^T dt' L \right) \delta(M(t_0) = M_0) \delta(M(T) = M_T),$$

$$\underline{DM} = \lim_{u \rightarrow \infty} \prod_{\rho=1}^{u+1} g^{1/2} \prod_G (2\pi\theta)^{-1/2} dM_\rho^G,$$

$$L(\dot{M}^G, M^G, t) = \frac{1}{2} (\dot{M}^G - h^G) g_{GG'} (\dot{M}^{G'} - h^{G'}) + \frac{1}{2} h^G_{,G} + R/6 - V,$$

$$[\cdots]_{,G} = \frac{\partial[\cdots]}{\partial M^G},$$

$$h^G = g^G - \frac{1}{2} g^{-1/2} (g^{1/2} g^{GG'})_{,G'},$$

$$g_{GG'} = (g^{GG'})^{-1},$$

$$g = \det(g_{GG'}),$$

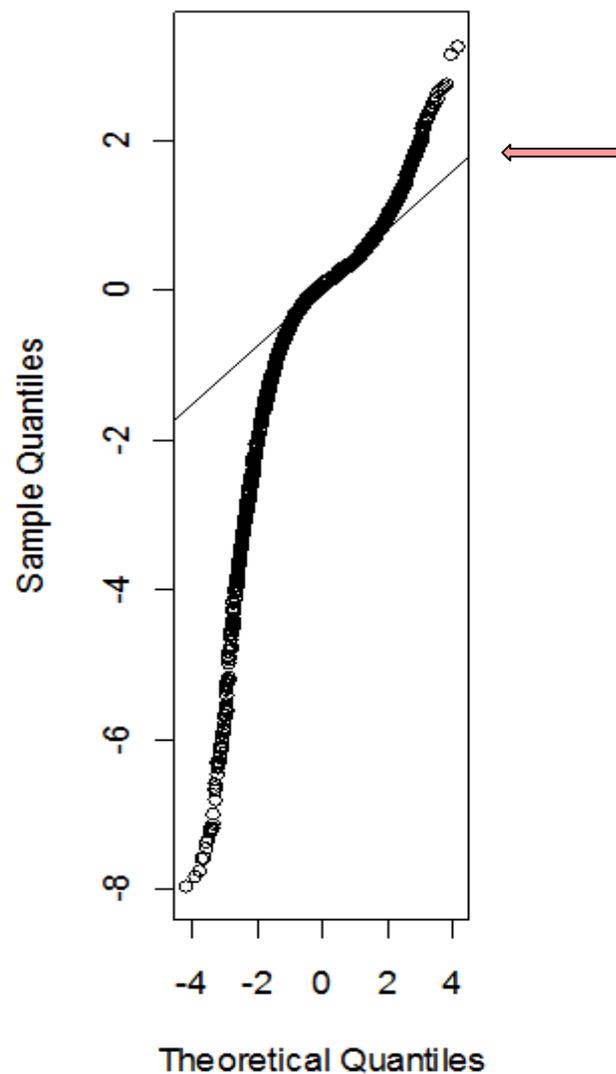
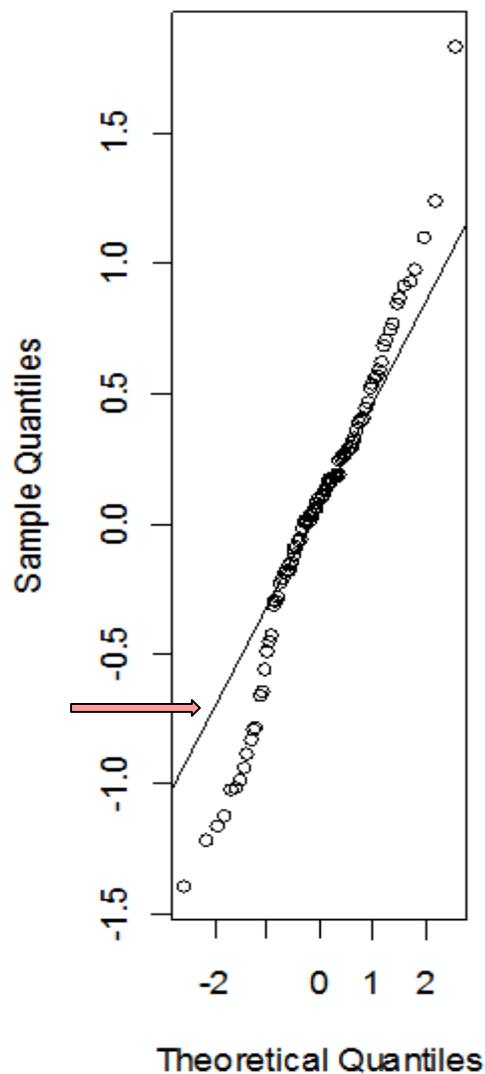
$$h^G_{,G} = h^G_{,G} + \Gamma_{GF}^F h^G = g^{-1/2} (g^{1/2} h^G)_{,G},$$

$$\Gamma_{JK}^F \equiv g^{LF} [JK, L] = g^{LF} (g_{JL,K} + g_{KL,J} - g_{JK,L}),$$

$$R = g^{JL} R_{JL} = g^{JL} g^{JK} R_{FJKL},$$

$$R_{FJKL} = \frac{1}{2} (g_{FK,JL} - g_{JK,FL} - g_{FL,JK} + g_{JL,FK}) + g_{MN} (\Gamma_{FK}^M \Gamma_{JL}^N - \Gamma_{FL}^M \Gamma_{JK}^N).$$

Plot, Sample of 100 Yearly Q-Q Plot, All Annual Return



- Numerical solutions of **pricing equations**: finite differences, finite elements, and spectral techniques in one and multiple dimensions
- Simulation approaches in **pricing and risk management**: advances in Monte Carlo and quasi-Monte Carlo methodologies; new strategies for market factors simulation.
- Optimization techniques in **hedging and risk management**

- Fundamental **numerical analysis** relevant to finance: effect of boundary treatments on accuracy; new discretization of time-series analysis
- Developments in free-boundary problems in finance: alternative ways and numerical implications in **American option pricing**

## Journal of Computational Finance



Focusing on the advances in numerical and computational techniques in pricing, hedging and risk management of financial instruments

## Journal of Credit Risk

Focuses on the measurement and management of credit risk, and the valuation and hedging of credit products, with the aim of promoting a greater understanding in credit risk theory



## Journal of Energy Markets

A major research outlet for new empirical and model based work in energy markets, dealing with the evolution and behaviour of electricity



## Journal of Financial Market Infrastructures



The first journal to focus on the emerging field of financial market infrastructures; analysing and furthering the development of this exciting sector

## Journal of Investment Strategies

Putting you at the forefront of modern investment strategies, the journal meets the thirst for fresh views on this crucial discipline



## Journal of Network Theory In Finance

An interdisciplinary journal publishing academically rigorous, practitioner focused research on the application of network theory in finance



## Journal of Operational Risk



Stimulating active discussions of practical approaches to quantify, model and manage operational risk

## Journal of Risk

The only publication devoted exclusively to theoretical and empirical studies in financial risk management. The journal promotes far-reaching research on the latest innovations, with particular focus on the measurement, management and analysis of financial risk



## Journal of Risk Model Validation

Focuses on the implementation and validation of risk models, and aims to provide a greater understanding of the key issues



- “Dispersion of possible outcomes” –P. Jorion (father of VaR)
- “Risk is that in the long run, returns will be terrible”
- "If you put a gun to my head and asked me what my firm's risk was, I would use VaR." -Richard Bookstaber, hedge-fund manager
- “In peacetime, you think about other people's intentions, In wartime, only their capabilities matter. VaR is a peacetime statistic" -Aaron Brown, Risk Manager at AQR, formerly Morgan Stanley.

- Investments analysis
- stochastic modeling for markets and finance
- simulation-based and quantitative portfolio selection and management
- display of quantitative information
- improved communication
- applications of engineering models to other statistical problems

- Academic finance tells you the best you can do is invest in the total market



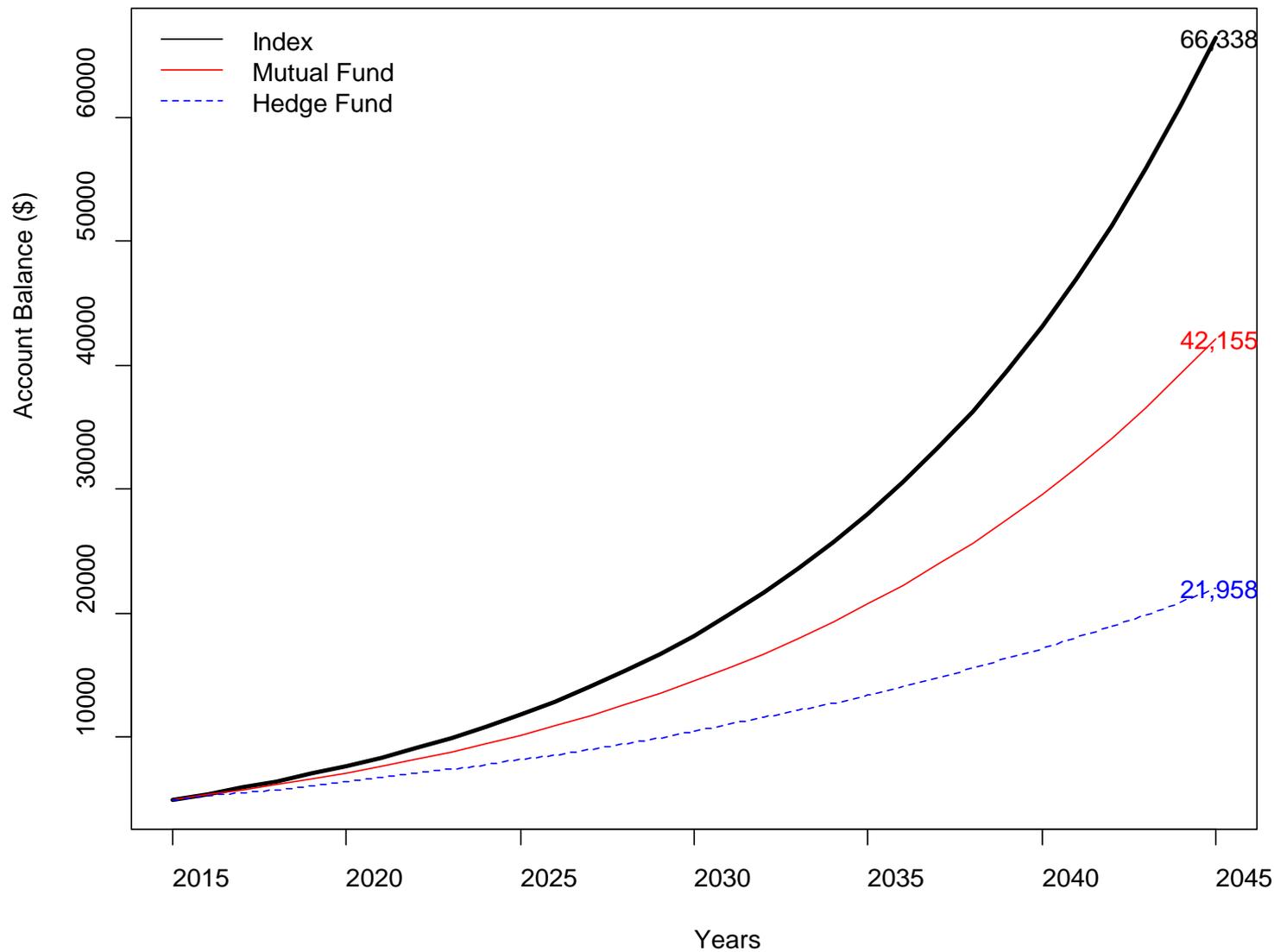
1/1/85 DJIA=1284; 6/9/17 DJIA = 21,272

Number of years: 32.5 Years

Total Return =  $21272/1284 = 16.57 \rightarrow r^* = 9\%$

- Over very long periods (91 years) the major **stock indexes** return about 6% (excluding dividends); lately (32 years) it's been 9%
- **Mutual funds** claim to do better
  - Charge 1% to 1.5%
- **Hedge funds** claim to do better
  - Charge 2% of principal + 20% of gains (when there are gains)

## Comparison of Investment Vehicles Nest Egg = \$5,000

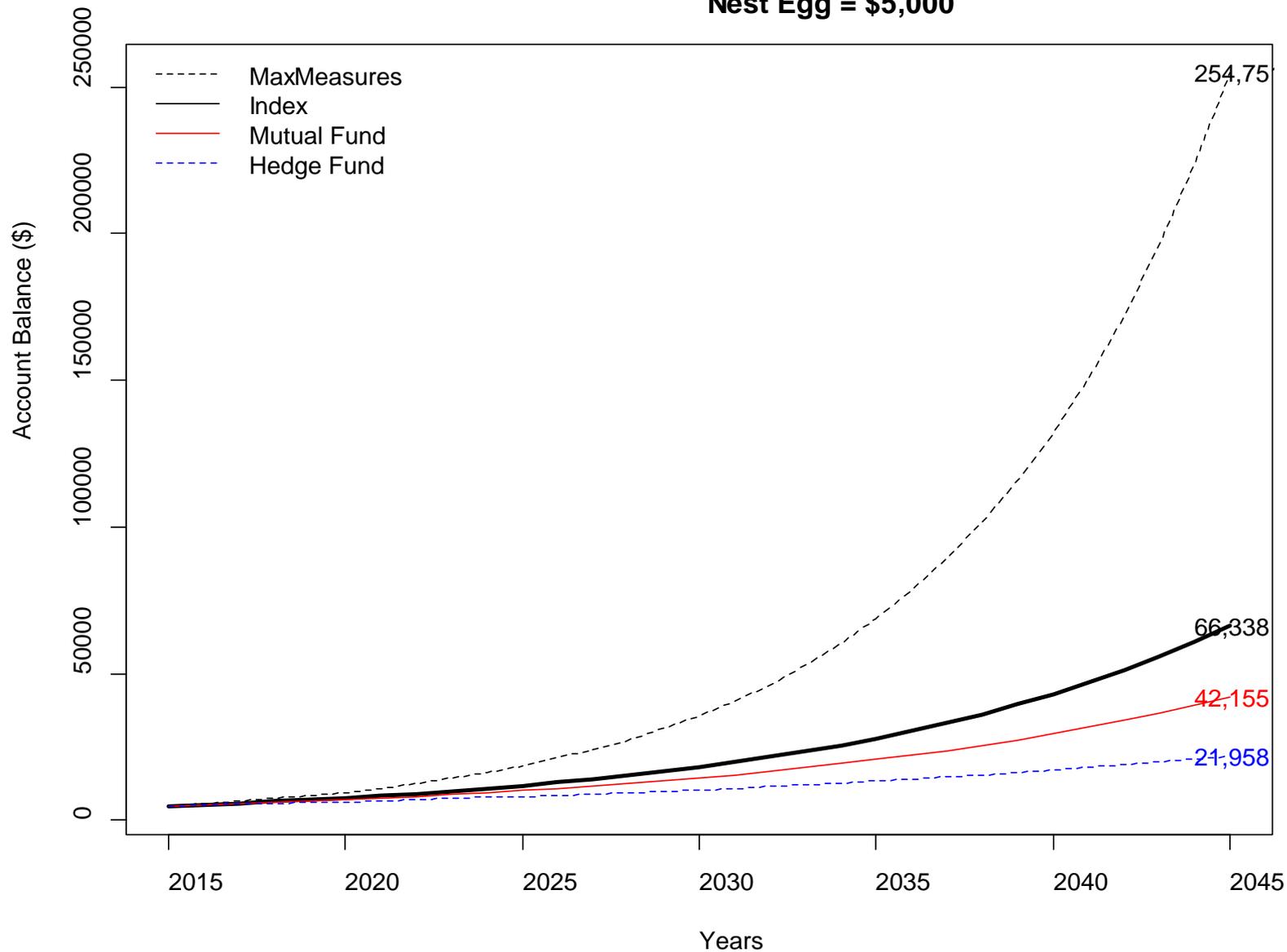


Current Date:		12/31/16	CAGR			
CRSP Begin Dates		N	vwretd	vwretx	ewretd	ewretx
SP500	12/31/1925	91	0.099	0.057	0.117	0.077
NYSE	12/31/1925	91	0.098	0.056	0.123	0.081
AMEX	12/31/1962	54	0.078	0.054	0.128	0.105
NASDAQ	12/31/1972	44	0.102	0.086	0.129	0.115

- Max Median Approach (excluding divs)
  - CAGR 1958-2015: 8.6%
  - Exceeds EWX S&P 500 with only 20 stocks
- MaxMeasures (Including dividends)
  - Lookback – 5 months, harmonic mean
  - CAGR 1970-2011: 14%

## Comparison of Investment Vehicles

Nest Egg = \$5,000



Actual DJIA vs. Idealized  
(Day In, Day out for 32 years)

