

Problem 1

15 min

Test 2 Solutions
(PRACTICE) 11

1. Which of the following are statistics?

- ☐ a. All of these choices. ~~X~~
- ☒ b. median
- ☒ c. mean ~~pop par~~
- ☐ d. standard deviation
- ☐ e. the population mean
- ☐ f. the population standard deviation
- ☒ g. the sample proportion
- ☐ h. the population proportion, π

2. What is the distribution of the values of a statistic called?

- ☐ a. a sample
- ☒ b. a sampling distribution
- ☐ c. Central Limit Theorem
- ☐ d. a mean

3. Which of the following defines the sampling distribution of \bar{x} ?

- ☒ a. It is the distribution of the sample means for all samples of the same size from the population.
- ☐ b. It is the probability distribution of a population mean.
- ☐ c. It is the probability distribution of the sample proportion based on a random sample of size n .
- ☐ d. It is the probability distribution of the sample means for all possible sample sizes from the population.

4. When can the Central Limit Theorem be safely applied using the conservative rule?

Multiple Guess 1

- 1 skip
- 2 b
- 3 a
- 4 a
- 5 b
- 6 b
- 7 a
- 8 c
- 9 b
- 10 c
- 11 d
- 12 b
- 13 c

Multiple Guess 2

- 1 a
- 2 b
- 3 b
- 4 c
- 5 d
- 6 a
- 7 c
- 8 skip (a)
- 9 d
- 10 a
- 11 b
- 12 c
- 13 skip (a)

Misleading

Problem 1

- ☒ a. When n is greater than 30
- ☐ b. When n is greater than or equal to 10
- ☐ c. When n is less than 10
- ☐ d. When n is greater than 20

5. Suppose a random sample of 15 snow throwers has a mean lifespan of 20 years. If it is known that $\sigma = 4$, what is the test statistic for testing $H_0: \mu = 18$ versus $H_a: \mu < 18$?

- ☐ a. 0.026
- ☒ b. $z = -1.94$
- ☐ c. -0.50
- ☐ d. $t = 1.94$ with $df = 14$

$$z = +1.94$$

$$T = Z = \frac{\bar{X} - 18}{4/\sqrt{15}} = \frac{20 - 18}{1.033} = 1.94$$

~~H_a is 1-sided to left so answer can only be b since C is wrong.~~

6. What is the probability of a Type I error of a test level of significance is used for the test? $H_0: \pi = 0.62$ versus $H_a: \pi \neq 0.62$ of if a 0.05

- ☐ a. 0.62
- ☒ b. 0.05
- ☐ c. The sample data is needed to determine the probability of a Type I error for a test.
- ☐ d. 0.10

This prob should have taken less 1 second, plus 1 second to check.

7. Suppose 15 students pass an exam in a class of size 25. If the population proportion of students who pass the exam is 0.65, what are the mean and standard deviation for the sampling distribution of \hat{p} ?

- ☒ a. $\mu_{\hat{p}} = 0.65, \sigma_{\hat{p}} = 0.095$
- ☐ b. $\mu_{\hat{p}} = 0.05, \sigma_{\hat{p}} = 0.0455$
- ☐ c. $\mu_{\hat{p}} = 0.60, \sigma_{\hat{p}} = 0.095$
- ☐ d. $\mu_{\hat{p}} = 0.60, \sigma_{\hat{p}} = 0.098$

\hat{p} No.

$$X \sim \text{Binom}(25, 0.65)$$

$$\begin{aligned} \hat{p} &\sim N(p, \sqrt{pq/n}) \\ &= N(0.65, \sqrt{0.2175/25}) \\ &= N(0.65, 0.095) \end{aligned}$$

8. When are two population means considered identical?

- ☐ a. when $\bar{x}_1 = \bar{x}_2$
- ☐ b. when $\sigma_1 = \sigma_2$

$$\mu_1 = \mu_2$$

No Right Answer

$$\hat{p} = 0.6$$

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- ☒ c. when $\mu_1 = \mu_2$
- ☐ d. when $\mu_1 > \mu_2$

9. What is the test statistic for testing whether or not the true proportion of adults who visit a dentist regularly is indeed $\mu = 0.72$ or whether it is less? Suppose a random sample of 30 adults found that the proportion who visited a dentist regularly was $p = 0.67$.

- ☐ a. $z = 0.61$
- ☒ b. $z = -0.6099$
- ☐ c. $z = -0.5824$
- ☐ d. $z = 7.8044$

$$H_0: p = .72 = p_0$$

$$H_A: p < .72$$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{.67 - .72}{\sqrt{.72(.28) / 30}} =$$

$$\sqrt{p_0 q_0 / n} = \frac{.05}{.08} = .6099$$

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What is the P -value for a test of $H_0: \pi = 0.4$ versus $H_a: \pi \neq 0.4$ with a test statistic of $z = 1.8$?

- ☐ a. 1.928
- ☐ b. 0.9641
- ☒ c. 0.072
- ☐ d. 0.036

2 sided

$$z = \frac{\hat{p} - .4}{\sqrt{\text{whatewr}}} = 1.8$$


$p\text{val} = 2 \cdot \Phi(z) = .072$

11. Which of the following is not true with regards to a P -value?

- ☐ a. A P -value indicates the strength of the evidence against the null hypothesis. ✓ T
- ☐ b. A P -value does not tell us the probability that the null hypothesis is true. T
- ☐ c. A P -value is a probability and must be between 0 and 1. ✓ T
- ☒ d. The larger the P -value the more conclusive the evidence is against the null hypothesis. F

Small p -value \Rightarrow reject H_0

12. If a computer is not available, what is a conservative estimate of the number of degrees of freedoms for the t curve when computing a two-sample confidence interval for the difference

between two independent population means if $n_1 = 40$ and $n_2 = 20$?

- ☐ a. 60
- ☒ b. 19
- ☐ c. 39
- ☐ d. 20

$$d.f. = \min(n_1, n_2) - 1 = 19$$

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13. The following *Minitab* output shows a comparison in yield for two types of wheat planted by Farmer Fred. What can you conclude from the information given?
Two-sample T for Yield

Field	N	Mean	StDev	SE Mean
1	6	2.940	0.303	0.12
2	6	2.960	0.373	0.15

all the
√n stuff is
here

bushels/acre?

Difference = $\mu_1 - \mu_2$

Estimate for difference: -0.020000

95% CI for difference: (-0.463483, 0.423483)

T-Test of difference = 0 (vs not =): T-Value = -0.10 P-Value = 0.921 DF = 9

Close to zero.

Practically
a. There is no evidence to support a difference in the yield of the two types of wheat.

b. The hypotheses being tested are $H_0: \mu_1 - \mu_2 = 0$ versus $H_a: \mu_1 - \mu_2 > 0$ Can't tell.

c. The test statistic is $z = -0.10$. No, t statistic

d. A 90% confidence interval for the difference in the mean yields is (-0.46, 0.42).

NO!

25 minutes

1. Fill in the blank. An unbiased statistic is a statistic whose mean value is equal to the population characteristic estimated.

- ☒ a. equal to
- ☐ b. half the value of
- ☐ c. greater than
- ☐ d. less than

2. What is the confidence level associated with an interval for estimating π that has the form

$$\left(p - 1.96 \sqrt{\frac{p(1-p)}{n}}, p + 1.96 \sqrt{\frac{p(1-p)}{n}} \right) ?$$

$1.96 \rightarrow 0.025$

- ☐ a. 5%
- ☒ b. 95%
- ☐ c. 90%
- ☐ d. 1.96

3. Consider the Minitab output given below. What is the value for the sample proportion used to compute the 95% confidence interval?

Sample	X	N	Sample p	95% CI
1	38	70	???	(0.419421, 0.662552)

$p \in \hat{p} \pm 1.96 \cdot SE_{\hat{p}}$

- ☐ a. 0.6625
- ☒ b. 0.542857
- ☐ c. 38
- ☐ d. 0.4194

Symmetric find midpt = \hat{p}

$$.6625 - .4194 = .2431 \rightarrow \frac{.2431}{2} = .12155$$

split diff

$$.6625 - .12155 = .54095$$

$$.4194 + .12155 = .54095$$

OR - (duh!) $\hat{p} = 38/70 = .542857!$

4. What sample size should be used if we would like to estimate the mean age of the college students at a particular campus with 99% confidence? We would also like to be accurate within 3 years and we will assume the population is normally distributed with a standard deviation of 4.5 years.

- ☐ a. A sample size of at least 14 should be used.
- ☐ b. A sample size of at least 9 should be used.

$X \sim N(\mu, 4.5)$

$\bar{X} \sim N(\mu, \frac{4.5}{\sqrt{n}})$

Marginal error = M

99% C.I. = $\bar{X} \pm z_{\frac{.01}{2}} \cdot \frac{4.5}{\sqrt{n}}$

$M = 3 = 2.57 \left(\frac{4.5}{\sqrt{n}} \right) \rightarrow \sqrt{n} = 3.86$
 $n = 14.86$

8 min

$z_c = 2.576$

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- ☒ c. A sample size of at least 15 should be used.
- ☐ d. A sample size of at least 26 should be used.

5. How are t distributions distinguished from one another?

- ☐ a. their standard deviation
- ☐ b. All t distributions are exactly the same. NO!
- ☐ c. their mean
- ☒ d. their degrees of freedom ✓

6. What is a point estimate for the population mean of GPA based on the Minitab output below from a random sample of data from the population?

One-Sample T: GPA

Variable	N	Mean	StDev	SE Mean	99% CI
GPA	200	2.63000	0.58033	0.04104	(2.52328, 2.73672)

$\mu \in 99\text{C.I.}$

- ☒ a. 2.63
- ☐ b. (2.52328, 2.73672)
- ☐ c. 0.58
- ☐ d. 200

point est.
interval

7. Which of the following is the test statistic for a hypothesis of a population mean if the population standard deviation is unknown?

→ going to be a t !

- ☐ a. $z = \frac{\bar{x} - \mu_{\text{hypothesized}}}{\frac{\sigma}{\sqrt{n}}}$
- ☐ b. $t = \frac{\bar{x} - \mu_{\text{hypothesized}}}{\frac{\sigma}{\sqrt{n}}}, df = n - 2$
- ☒ c. $t = \frac{\bar{x} - \mu_{\text{hypothesized}}}{\frac{s}{\sqrt{n}}}, df = n - 1$ ✓

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d. $z = \frac{p - \pi_{\text{hypothesized}}}{\sqrt{\frac{\pi_{\text{hypothesized}}(1 - \pi_{\text{hypothesized}})}{n}}}$

Not proportion!

8. Which of the following is the correct formula for constructing a confidence interval for μ when σ is unknown and either the sample size is large or the population distribution is normal?

a. $\bar{x} \pm (t \text{ critical value}) \frac{s}{\sqrt{n}}$

\rightarrow prob. would have 'oh

b. $p \pm (z \text{ critical value}) \sqrt{\frac{p(1-p)}{n}}$

c. $\bar{x} \pm (z \text{ critical value}) \frac{\sigma}{\sqrt{n}}$

d. $\mu \pm (z \text{ critical value}) \frac{\sigma}{\sqrt{n}}$

} σ unknown.

Confusing

$\mu \in \bar{X} \pm z_c \cdot SE_{\bar{X}}, \sigma \text{ unknown.}$

Problem 2

only signif @ 10% level.

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9. Suppose the P-value equals 0.09 for testing whether grocery stores stocked on average more than 30 varieties of potato chips. Which of the following conclusions would be correct for testing the

$H_0: \mu = 30$ versus $H_a: \mu > 30$ hypotheses?

$$H_0: \mu = 30$$

$$H_a: \mu > 30$$

- ☐ a. It can be concluded that stores stock more than 30 varieties on average using the $\alpha = 0.01$ significance level. *NO*
- ☐ b. It can be concluded that stores stock more than 30 varieties on average using the $\alpha = 0.05$ significance level. *NO*
- ☐ c. It can be concluded that on average stores ~~do not~~ stock more than 30 varieties using the $\alpha = 0.10$ significance level.
- ☒ d. It can be concluded that on average stores do not stock more than 30 varieties using the $\alpha = 0.05$ significance level.

Weasel-worded but only choice

10. Which of the following is the correct null hypothesis for testing whether two population means are the same?

- ☒ a. $H_0: \mu_1 - \mu_2 = 0$ ✓
- ☐ b. $H_0: \bar{x}_1 - \bar{x}_2 = 0$ ← *No,*
- ☐ c. $H_0: \mu_1 - \mu_2 \neq 0$ *Alternative*
- ☐ d. $H_0: \mu_1 - \mu_2 = 1$ *wierd*

11. If two independent random samples gave the following information, what would be the t value for testing that the population means are identical? Assume the populations are approximately normal.

Sample A: $n_A = 10, \bar{x}_A = 10, s_A = 5$

Sample B: $n_B = 20, \bar{x}_B = 12, s_B = 6$

- ☐ a. $t = 2.0$
- ☒ b. $t = -0.96$
- ☐ c. $t = 5.79$
- ☐ d. $t = -2.24$

$$t = \frac{\Delta \bar{X} - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{-2}{\sqrt{\frac{25}{10} + \frac{36}{20}}}$$

$$= -2 / \sqrt{4.3} = \frac{-2}{2.07}$$

$$= -0.96$$

$n = 6$

12. What can be concluded from the following *Minitab* output in a study the heights of six randomly chosen first graders at the beginning of the school year (September) and the end of the school year (June)?

Paired T for height in June - height in September

June-Sept : should be > 0 if grew.

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	N	Mean	StDev	SE Mean
height in June	6	45.2833	5.4609	2.2294
height in Septem	6	43.8333	5.4924	2.2423
Difference	6	1.45000	1.16404	0.47522

} include Tn stuff

95% lower bound for mean difference: 0.49241

T-Test of mean difference = 0 (vs > 0): T-Value = 3.05 P-Value = 0.014

- Since the mean difference is 1.45 we can conclude that students grown on average 1.45 inches per year and thus there is a difference between mean heights of the students at the beginning of the school year and the mean height of students at the end of the school year at the $\alpha=0.01$ level. $\times p=0.014$
- ~~The data support the theory that there is not a difference between mean heights of the~~
- ☐ b. students at the beginning of the school year and the mean height of students at the end of the school year at the $\alpha=0.05$ level.
- ~~The data support the theory that there is a difference between mean heights of the~~
- ☒ c. students at the beginning of the school year and the mean height of students at the end of the school year at the $\alpha=0.05$ level. \checkmark
- ~~The data support the theory that there is a difference between mean heights of the~~
- ☐ d. students at the beginning of the school year and the mean height of students at the end of the school year at the $\alpha=0.01$ level. $\times 0.014$

37 min so far \Rightarrow 12 min to do the last two...

Problem 2

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p_{common}

$$p_c = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

13. What is the formula

☒ a. It is the test statistic for comparing two population proportions.

☐ b. It is the standard deviation used when constructing a confidence interval for $\pi_1 - \pi_2$.

☐ c. It is the pooled standard deviation for two proportions.

☐ d. It is the statistic for estimating the common population proportion when $\pi_1 = \pi_2$.

No Way to remember this.

S.D. common p.

No!

\hat{p} . $z = (\hat{p}_1 - \hat{p}_2) / \sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}$ (in Hypo test Setting).

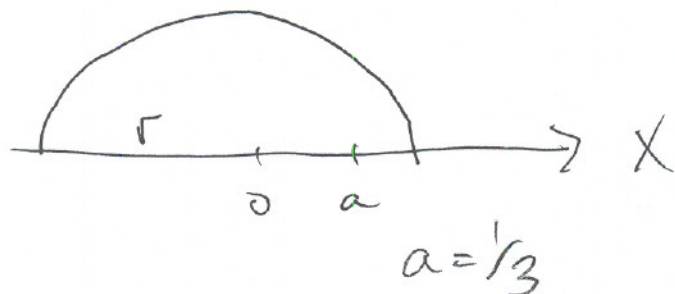
25 min \Rightarrow 40 minutes so far,

10 min. for Next two problems!

Thanks to
Anthony Macaluso.

X is a r.v. with pdf the semi-circle, i.e.,

$-r \leq X \leq r$, and the equation for the circle y is according to the classic formula for Circle, $r^2 = x^2 + y^2$



$$a = 1/3$$

HINT 1: Area of a chord @ $a =$

$$R^2 \tan^{-1} \left(\frac{\sqrt{(R/a)^2 - 1}}{1} \right) - r \sqrt{R^2 - r^2}$$

a. Calculate r

$$A = (\pi r^2) + .5 = 1.0$$

$$r = \sqrt{\frac{2}{\pi}} = .7979$$

b. Write an expression to evaluate $\Pr(X \leq a)$

$$x^2 + y^2 = r^2$$

$$y = \sqrt{r^2 - x^2}$$

$$A = \int_{-r}^a \sqrt{r^2 - x^2} dx \dots$$

HINT 2: $\int \sqrt{a^2 - x^2} =$

$$\frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

c. Find this probability.

Evaluate using handy Hin. — OR —



use the Chord Area equation, keeping proper scaling to Account for

