# Stat 310 Exam 1 Review Sheet

Let $P(A) = P(B) = 1/3$ and $P(AB) = 1/10$. Find

$$\begin{aligned}&P(B^c),\\&P(A \cup B^c),\\&P(A^cB),\\&P(A^c \cup B^c)\end{aligned}$$

A box contains five green balls, three black balls, and seven red balls. Two balls are selected randomly without replacement from the box. What is the probability that (a) both balls are red? (b) both balls are the same color?

Drawer $A$ contains five pennies and three dimes, while drawer $B$ contains three pennies and seven dimes. A drawer is selected at random, and a coin is selected at random from the chosen drawer. (a) Find the probability of selecting a dime. (b) Suppose a dime is obtained. What is the probability that it came from drawer B?

Let $P(A) = 0.4$ and $P(A \cup B) = 0.6$. (a) For what value of $P(B)$ are $A$ and $B$ mutually exclusive? (b) For what value of $P(B)$ are $A$ and $B$ independent?

The probability that a marksman hits a target is 0.9 on any given shot, and repeated shots are independent. He has two pistols; one contains two bullets and the other contains only one bullet. He selects a gun at random and fires until the gun is empty. What is the probability of hitting the target exactly one time? Given that the target is hit exactly once, what is the probability that his gun had two bullets?

A game consists of rolling a six-sided die once and then flipping a fair coin. The score is given by the number of spots on the die plus the number of heads showing on the coin toss. Let $X$ denote the value of the score. (a) Find the pmf of $X$. (b) Find the cdf of $X$. (c) What is $P(X > 3)$?

A nonnegative integer-valued random variable $X$ has a cdf of

$$F_X(x) = 1 - (1/2)^{x+1}, \quad x \geq 0.$$

(a) Find the probability mass function of $X$. (b) Find $P(10 \leq X \leq 20)$.

A continuous random variable $X$ has pdf given by $f_X(x) = c(1-x)x^2$ if $0 < x < 1$ and zero otherwise. (a) Find $c$. (b) Find $E(X)$.

Find the pdf corresponding to the following cdfs:

$$\begin{aligned}F_X(x) &= (x^2 + 2x + 1)/16; \quad -1 < x < 3\\F_X(x) &= 1 - e^{-\lambda x} + \lambda x e^{-\lambda x}; \quad 0 \leq x < \infty, \quad \lambda > 0.\end{aligned}$$

Let $X$ be continuous with pdf $f_X(x) = 3x^2$ if $0 < x < 1$, and zero otherwise. (a) Find $E(X)$. (b) Find $Var(X)$. (c) Find $E(X^k)$. (d) Find $E(3X - 5X^2 + 1)$.

From the integers $1, \ldots, 10$, three numbers are chosen at random without replacement and disregarding the order. (a) In how many ways can this be done? (b) What is the probability that all of the numbers are greater than or equal to 5? (c) What is the probability that the smallest number is 4?

Suppose a value $x$ is chosen "at random" in the interval $[0, 10]$. In other words, $x$ in an observed value of a random variable $X \sim U(0, 10)$. The value $x$ divides the interval into two subintervals. (a) Find the CDF of the length of the shorter subinterval. (b) What is the probability that the ratio of the lengths of the shorter to the longer subinterval is less than $1/4$?

Assume that the time (in hours) until failure of a transistor is a random variable $X \sim$ Exponential$(\lambda = 0.01)$. (a) Find the probability that $X > 110$. (b) It is observed after 95 hours that the transistor is still working. Find the conditional probability that $X > 110$. (c) Find the conditional mean of $X$, given that it was still working after 95 hours.

A certain assembly line produces electronic components, and defective components occur independently with probability 0.01. The assembly line produces 500 components per hour. For a given hour, what is the probability that the number of defective components is at most two?

Let $X \sim N(10, 4)$ and $Y \sim N(12, 9)$. Assume that $X$ and $Y$ are independent. Find $P(2X + 3Y \leq 50)$.

Let $X \sim Gamma(\alpha = 2, \lambda = 1)$ and $Y \sim Gamma(\alpha = 4, \lambda = 1)$. Assume that $X$ and $Y$ are independent. Find $f_{X+Y}$. Find $E(X + Y)$. Find $Var(X + Y)$.

Two friends have agreed to meet at 12:30. Assume that their arrival times are independent random variables, uniformly distributed between 12:30 and 1:00. Let $X$ and $Y$ denote the arrival times (in minutes after 12:30). (a) Sketch the region where $f_{XY} > 0$. (b) Shade in the region corresponding to the earlier arriver having to wait more than 10 minutes for the other one. (c) Compute the probability that the earlier arriver has to wait more than 10 minutes for the other one.

Three distinct integers are chosen at random without replacement from the first 20 positive integers. Compute the probability that (a) Their sum is even. (b) Their product is even.

Let $f_X(x) = 2x$, $0 < x < 1$. (a) Compute $f_Y(y)$, where $Y = \sqrt{X}$. (b) Compute $E(\sqrt{X})$.

The random variable $X$ has pdf $f_X(x) = 1/3$, $-1 < x < 2$, zero elsewhere. (a) Find the moment generating function of $X$. (b) Find the mean and variance of $X$. (c) Let $Y = X_1 + X_2 + X_3 + X_4$, where the $X_i$ are all independent and have common distribution $X$ as given above. Find the mgf, mean, and variance of $Y$.

The random variable $X$ is said to have a two-parameter Expoential$(\lambda, \eta)$ distribution if

$$f_X(x) = \lambda e^{-\lambda(x-\eta)}, \quad \eta < x < \infty, \quad \lambda > 0.$$

(a) Find the moment generating function of $X$. (b) Find the mean and variance of $X$. (c) Let $Y = X_1 + X_2 + X_3 + X_4$, where the $X_i$ are all independent and have common distribution $X$ as given above. Find the mgf, mean, and variance of $Y$.

Let $X$ and $Y$ have the joint pdf

$$f_{XY} = 21x^2y^3, \quad 0 < x < y < 1.$$

(a) Find the marginal density of $X$. Careful with the limits of integration! A sketch may help. (b) Find the conditional density of $Y$ given $X$. (c) Find $E(Y|X)$.

Let $X \sim \text{Uniform}(0,1)$. Find $f_Y(y)$, where $Y = -2\log(X)$.

Let $X_1$, $X_2$ and $X_3$ be independent with common pdf

$$f_X(x) = 3x^2, \quad 0 < x < 1.$$

Find the probability that exactly two of these three random variables exceed 1/2.

Let $f_X(x) = x^2/9$, $0 < x < 3$, zero elsewhere. Find $f_Y(y)$ when $Y = X^3$.

$X_1$ and $X_2$ are independent normal random variables with common distribution $N(\mu_X, \sigma_X^2)$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + 2X_2$. Find $Corr(Y_1, Y_2)$.

Let $X_1, \ldots, X_{100}$ be a random sample from an Exponential distribution with $\lambda = 1$. Let $Y = \sum_{i=1}^{100} X_i$. (a) Using the Central Limit Theorem, give an approximation for $P(Y > 110)$. (b) If $\bar{X}$ is the sample mean, approximate $P(1.1 < \bar{X} < 1.2)$.

A student is taking a multiple-choice exam with 10 questions on it. There are 4 possible answers for each question, and the student is able to rule out one of the four in each case, but must guess amongst the remaining 3. What is the probability that the student gets at least 3 questions right? Let $Y$ be the number of questions the student gets right. What is the distribution of $Y$? What are the parameter(s)?