Stat 331/Elec 331, Homework 3, October 2

Solutions should be clear and easy to follow. You are allowed to use the book and lecture notes. Boldface numbers within parentheses denote the maximum score on each problem.

Solutions are due on the date at the top. If you can not come to class and hand it to me there, you will have to come by my office (slide it under the door if I am not there). If you can not make it on time, you may still return your solutions but there will be a two point deduction for each day you are late.

1. Let $R \sim$ unif $[0, 1]$ and let V be the volume of a sphere with radius R. Compute $E[V]$ and Var$[V]$. (**3**)

2. If X is a continuous random varible, the *median* of X is defined as the value m which is such that $P(X \leq m) = P(X \geq m) = 1/2$, in some sense the "midpoint" of the distribution. In the following cases, find the median m and compare with the mean μ.

a. $X \sim$unif$[a, b]$.

b. $X \sim$exp(a).

c. X has pdf $f(x) = 1/x^2, \ x \geq 1$. (**3**)

3. The non-negative continuous random variable X has failure rate function $h(t) = 1/(1 + t), t \geq 0$. Find the pdf of X (**3**)

4. Let $X \sim$ unif$(0, 1)$. Find the failure rate function $h(t)$ of X. What happens to $h(t)$ as $t \to 1$? Explain intuitively. (**3**)

5a. Let X be a discrete random variable with range $\{1, 2, ...\}$. The (discrete) failure rate function is then defined as

$$r(k) = \frac{P(X = k)}{P(X \geq k)}.$$

Show that $r(k) = P(X = k | X \geq k)$.

b. Let X be the number of dots when you roll a fair die. Find the failure rate function of X. the pmf and the failure rate function of X and explain the difference between the two graphs. (**4**)