Stat 331, Homework 4, October 30

Solutions should be clear and easy to follow. You are allowed to use the book and lecture notes. Boldface numbers within parentheses denote the maximum score on each problem.

Solutions are due on the date at the top. If you can not come to class and hand it to me there, you will have to come by my office (slide it under the door if I am not there). If you can not make it on time, you may still return your solutions but there will be a two point deduction for each day you are late.

1. Let (X, Y) be uniformly distributed on the triangle with corners in $(0, 0), (0, 1)$ and $(1, 0)$.

a. Compute the correlation coefficient ρ_{XY}. To save you some time and computations: $E[X] = E[Y] = 1/3$ and $E[X^2] = E[Y^2] = 1/6$.

b. If you have done **a** correctly, the value of ρ_{XY} is negative. Explain intutitively. (**3**)

2. A company manufactures metal plates of size 5 x 10 (inches). Due to random fluctations, a randomly selected such plate has a size of X x Y inches where (X, Y) follows a bivariate normal distribution with means 5 and 10, variances 0.01 and 0.04 and correlation coefficient 0.8.

a. A plate is useless if its circumference is less than 29 or more than 31 inches. What is the probability that this happens?

b. If the sidelength X of a plate is measured to be 5.1 inches, what is the probability that the plate is useless?

c. One way to improve the process is to try to reduce the variances of X and Y. Suppose that we can calibrate the process so that both variances are reduced by a factor c (so that X has variance $0.01c$ and Y $0.04c$). To get the probability in **a** down below 0.01, how small must c be? (**6**)

3. Let (X, Y) have a bivariate normal distribution. Show that X and Y are independent if and only if they are uncorrelated. (**3**)

4. Consider a certain phone line. When it is free, the waiting time W until the next incoming call has an exponential distribution with mean 1 minute. The time C it takes to complete a phone call has a uniform distribution with mean 1/2 minute. Consider a time point when the line is free and let T be the time until the next incoming call is completed. Find the pdf of T and sketch its graph. (**3**)