Stat 331, Homework 6, November 18

Solutions should be clear and easy to follow. You are allowed to use the book and lecture notes. Boldface numbers within parentheses denote the maximum score on each problem.

Solutions are due on the date at the top. If you can not come to class and hand it to me there, you will have to come by my office (slide it under the door if I am not there). If you can not make it on time, you may still return your solutions but there may be a two point deduction for each day you are late.

1. Suppose that we have a sample $X_1, ..., X_n$ on a uniform distribution on (a, b) where both a and b are unknown. Find the method of moments estimators of a and b. *Hint:* It's probably easiest to express the mean and variance both in terms of μ_1 and μ_2, and in terms of a and b. (**4**)

2. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with density

$$f_\theta(x) = \theta x^{\theta-1}, 0 \leq x \leq 1$$

where θ is an unknown parameter.

a. Find the maximum likelihood estimator of θ.

b. Show that the method of moments estimator of θ is $\hat{\theta} = \bar{X}/(1-\bar{X})$ where $\bar{X}$ is the sample mean.

c. Describe how to use the inverse transformation method to simulate observations from this distribution for a fixed value of θ.

d. It is hard to find exact expressions for the mean and variance of $\hat{\theta}$ from **b** but they can be investigated by simulation. Choose $\theta = 2$ and simulate a large number of samples $X_1, ..., X_n$ to estimate the mean and variance of $\hat{\theta}$. Also plot a histogram of $\hat{\theta}$ (note that one sample gives you one value of $\hat{\theta}$). Do this for sample sizes $n = 3, 10$ and 30. Note how the estimator does not seem to be unbiased, which should be seen clearly for $n = 3$, but how it gets closer to being unbiased as the sample size increases and also how the variance decreases, indicating that the estimator is consistent. (**8**)

3. Let $X_1, X_2, ..., X_n$ be a random sample from a normal distribution with mean 0 and unknown variance θ.

a. Find the maximum likelihood estimator of θ. Is it unbiased?

b. Find the method of moments estimator of θ. Is it unbiased? (**4**)