Stat 331/Elec 331, Midterm Exam, due October 9, at 5 pm

Solutions should be clear, complete and easy to follow. You are allowed to to use the book, lecture notes and previous homework with solutions. Collaboration is not allowed. The time limit is five hours. The maximum score is given at the end of each problem. Late turn-ins are not accepted.

1. The function f is defined as $f(x) = cx^2, \ 0 < x < 1$ (and 0 otherwise).

a. Determine the constant c so that this becomes a pdf for a continuous random variable X.

b. Compute $E[X]$, $Var[X]$ and $P(X > 0.5)$.

c. Let $Y = \sqrt{X}$ and find the pdf for Y.

d. Compute $E[Y]$ and $Var[Y]$. **(4)**

2. Are the following claims true or false? Give proofs or counterexamples.

a. If A is an event such that $P(A) = 0$, then $A = \emptyset$.

b. If p is the pmf for a discrete random variable then $p(x) \leq 1$ for all x.

c. If f is the pdf for a continuous random variable then $f(x) \leq 1$ for all x.

d. If $X \sim \exp(\lambda)$, then $2X \sim \exp\ (\lambda/2)$

e. If $X \sim$ unif $(-1, 1)$, then $|X| \sim$ unif $(0, 1)$.

f. If $X \sim$ bin (n, p), then $2X \sim$ bin $(2n, p)$. **(6)**

3. A transmittor sends 0's and 1's to a receiver. Each digit is received correctly (0 as 0, 1 as 1) with probability 0.90. Digits are received correctly independently of each other and on the average twice as many 0's as 1's are being sent.

a. If 1 is received, what is the probability that 1 was sent?

b. If the sequence 10 is sent, what is the probability that it is received incorrectly?

c. If the sequence 10 is received, what is the probability that this is the sequence that was sent? (**4**)

4. Two friends have agreed to meet at 12.30. Assume that their arrival times are independent random variables, one uniformly distributed between 12.30 and 1.00 and the other uniformly distributed between 12.30 and 1.15. Compute the probability that the one who arrives first must wait more than ten minutes.
(**4**)

5. Let X have a uniform distribution on $(0, 1)$ and given that $X = x$, let the conditional distribution of Y be uniform on $(0, 1/x)$.

a. Find the joint pdf $f(x, y)$ and sketch the region where it is positive.

b. Compute $P(X > Y)$.

c. Find $f_Y(y)$, the marginal pdf of Y, and sketch its graph. (**6**)