Stat 331, Solutions to Homework 7

1a. The possible states are 0,1,2, and 3. The rates are:

$$0 \to 2 : 1$$

$$1 \to 0 : 1, \quad 1 \to 3 : 1$$

$$2 \to 1 : 1$$

$$3 \to 2 : 1.$$

Solving the balance equations gives stationary distribution

$$\pi_0 = 1/5, \ \pi_1 = 1/5, \ \pi_2 = 2/5, \ \pi_3 = 1/5.$$

b. Once the system leaves 2, it goes through the cycle $1 \to 0 \to 2$ or $1 \to 3 \to 2$. In either case T is the sum of the holding times in 1 and 0 so $E[T] = 2$.

2a. If $N = 0$ then $Q = 0$. Otherwise, $Q = N - 1$.

b. If S is a service time, then $T = W + S$.

c. W is neither discrete nor continuous. Note that $W = 0$ if the system is empty which has probability $\pi_0 = 1 - \rho$. If the system is non-empty with n customers say, then W is the sum of n service times. Hence W has a discrete part in 0 and a continuous part on $(0, \infty)$. The cdf of W has a jump of size $1 - \rho$ at 0 and is continuous on $(0, \infty)$. Compare HW2, problem 5.

3a. The possible states are 0,1, and 2. If the system is in state 1, the stream of joining customers is the superposition of two thinned Poisson processes, one with rate $2 \cdot 0.5 = 1$, the other with rate $1 \cdot 0.8 = 1.8$. The service rate is 0.5 (mean $1/\beta = 2$). The rates are thus

$$0 \to 1 : 3$$

$$1 \to 2 : 1.8$$

$$1 \to 0 : 0.5$$

$$2 \to 1 : 0.5.$$

Solve the balance equations to get stationary distribution $\pi_0 = 5/143 = 0.035, \pi_1 = 30/143 = 0.210$ and $\pi_2 = 108/143 = 0.755$.

b. A caller is lost if the system is full, which has probability π_2 or if the system is in state 1, which has probability π_1 and the customer does not join which has probability 0.5 for men and 0.8 for women. Since there are on the average twice as many men as women, the probability that a caller is male is 2/3 and female 1/3. The probability to lose a caller is thus $\pi_2 + \pi_1((2/3) \cdot 0.5 + (1/3) \times 0.2) = \pi_2 + 0.4\pi_1 = 120/143 = 0.84$, i.e. 84% of the callers are lost.