STAT 331 Homework 3

Due date: In class on Thursday, September 23rd, 2004

Instructor: Dr. Rudolf Riedi

9. (10 points) Compute the mean and the variance of the geometric distribution (each 5 points). Recall that the geometric PMD is given by $p(k) = p^{k-1}q$ $(k = 1, 2, \ldots)$ with $q = 1 - p$.

10. (7 points) Researchers study the connection between a certain virus and gender in two different types of mice. For a population of mice A the study reveals the following chances:

$$\begin{aligned} P_{\mathrm{A}}[\text{virus and female}] &= 1/6 \\ P_{\mathrm{A}}[\text{no virus and female}] &= 1/2 \\ P_{\mathrm{A}}[\text{virus and male}] &= 1/12 \\ P_{\mathrm{A}}[\text{no virus and male}] &= 1/4 \end{aligned}$$

and for mice of type B:

$$\begin{aligned} P_{\mathrm{B}}[\text{virus and female}] &= 1/12 \\ P_{\mathrm{B}}[\text{no virus and female}] &= 7/12 \\ P_{\mathrm{B}}[\text{virus and male}] &= 1/6 \\ P_{\mathrm{B}}[\text{no virus and male}] &= 1/6 \end{aligned}$$

   (a) (3 points) For both types of mice used in this study compute the relative percentages of females and males, as well as the percentages of animals with or without the virus.

   (b) (2 points) For computational purposes, the following random variables are introduced: For mice type A, U_A indicates gender meaning that $U_A = 2$ for females, and $U_A = 1$ for males; similarly, $V_A = 1$ for animals with the virus, $V_A = 0$ for mice without. For mice type B the variables U_B and V_B are defined in analogous manner. The tables above provide the joint PMDs, e.g., $P[U_B = 2, V_B = 0] = 7/12$. Compute the marginals of all four r.v.

   (c) (2 points) For which type of mice are gender and health independent, meaning that the r.v. U and V are independent?

11. (10 points) Compute the mean and the variance of the Poisson distribution (each 5 points). Recall that the Poisson PMD is given by $p(k) = e^{-\lambda}\lambda^k/k!$ $(k = 0, 1, \ldots)$ with $\lambda > 0$, and that $\sum_k p(k) = 1$.

12. (10 points) An experiments consists of two parts. First an unfair coin with P[Heads] is flipped 3 times. The number K of Heads is noted. Second, a random number X is chosen from $\{0, \ldots, K\}$ uniformly, i.e., any number in $\{0, \ldots, K\}$ is equally likely to be drawn as X.

   (a) What is the distribution $p_K(\cdot)$ of K, and what is the conditional distribution of X given K, i.e., compute $p_{X|K=a}(b) = P[X = b|K = a]$.

   (b) Use (a) to compute the joint distribution of (K, X), i.e., compute $p_{KX}(a, b) = P(\{K = a\} \cap \{X = b\}]$.

13. (3 points) The random variable X takes the values $\{0, \ldots, 4\}$ all equally likely. Compute the probability mass distribution (PMD) of $Y = \sin(X \cdot \frac{\pi}{4})$.