STAT 331 Homework 4

Due date: In class on Thursday, September 30, 2004

Instructor: Dr. Rudolf Riedi

14. (10 points) We consider a binary channel, meaning a device which allows to transmit bits, i.e., zeroes and ones. Let X_k denote the bit sent in the kth transmission, and Y_k the bit received (or detected). We assume that the bits sent take the values 0 and 1 equally likely, i.e., $P[X_k = 1] = 1/2$. We also assume that the X_k are independent. The transmission of the considered binary channel might result in an error, which can be summarized as follows:

$$\begin{aligned} P[Y_k = 1 | X_k = 0] &= .05 \\ P[Y_k = 0 | X_k = 1] &= .10 \end{aligned}$$

Let $X = (X_1 X_2)$ be a transmitted 2-bit word. Let $Y = (Y_1 Y_2)$ be the received word. For simplicity, we identify the words with their numerical value, i.e., $(0,0) = 0$, $(0,1) = 1$, $(1,0) = 2$ and $(1,1) = 3$.

   (a) (5 points) Find the marginal distribution of Y, i.e., $p_Y(k)$ for $k = 0, 1, 2, 3$. ($p_Y(0) = .2756$)

   (b) (5 points) Let $E = X - Y$ be the numerical value of the error between Y and X. Find the expected value of E. (-0.0750)

15. (10 points) Two basketball players take turns in shooting from the 3-point range until one hits. Player A starts; he has a probability of 1/2 of hitting. Player B has a probability of .6 of hitting. Abbreviate by h the outcome of a shooter hitting, by m the outcome of missing. (Write h_A, h_B etc if confusion can arise.)

   (a) (5 points) Write down the event W of player A winning in terms of all possible sequences of hits and misses leading to A winning. Compute $P[W]$.

   (b) (5 points) Let $N|W$ be the number of shots A has to take for her to win, given that A wins. Compute the distribution of $N|W$, i.e., compute $p_{N|W}(k) = P[N = k|W]$. What type of distribution is this?

16. (10 points) An industrial customer purchases an expensive item that is critical to the company. If the item passes the acceptance test, then the customer pays the manufacturer \$10M (event A). If the item fails the acceptance test, the customer sues the manufacturer for \$5M (event B). If the manufacturer proves in court that the tests were defective and the item met specifications after all, the judge will award the manufacturer \$3M as a punitive judgement, plus the customer must accept and pay for the item (event (C). The engineers, statisticians and lawyers make the following estimates: $P[A] = .8$, $P[C|B]] = .6$.

   (a) (3 points) Construct a partition using the events A, B, and C. (There are (at least) two obvious partitions.)

   (b) (3 points) Find the probability $P[C]$ that the company pays the punitive judgement. (.12)

   (c) (4 points) Find the expected cost to the customer. (\$9.1M)

17. (10 points) Compute the characteristic function $\phi(u) = \mathbb{E}[e^{iuX}]$ for a r.v. X with geometric PMD: $p_X(k) = q^{k-1}p$ with $q = 1 - p$, $0 < q < 1$. (8 points) Verify that $\phi'(0) = i\mathbb{E}[X]$ using a formula obtained for $\mathbb{E}[X]$ in an earlier homework. (2 points)