ELEC 535 Homework 9

Due date: In class on Wednesday, April 16, 2003

Instructor: Rudolf Riedi Rice University, Spring 2003

Problem 9.1 (Mutual information for correlated normals.)

Find the mutual information $I(X;Y)$, where

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2\left(0, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix}\right).$$

Evaluate $I(X;Y)$ for $\rho = 1, \rho = 0$, and $\rho = -1$, and comment.

Problem 9.2 (Uniformly distributed noise.)

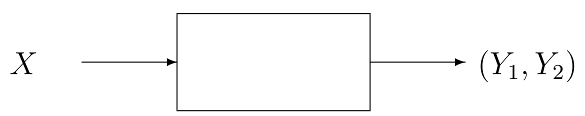
Let the input random variable X for a channel be uniformly distributed over the interval $-1/2 \leq x \leq +1/2$. Let the output of the channel be $Y = X + Z$, where the noise random variable is uniformly distributed over the interval $-a/2 \leq z \leq +a/2$.

1. Find $I(X;Y)$ as a function of a.
2. For $a = 1$ find the capacity of the channel when the input X is peak-limited; that is the range of X is limited to $-1/2 \leq x \leq 1/2$. What probability distribution on X maximizes the mutual information $I(X;Y)$?

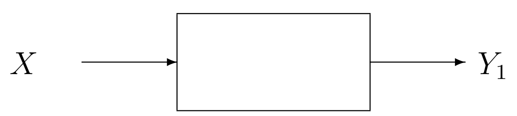
Problem 9.3 (A channel with two independent looks at Y.)

Let Y_1 and Y_2 be conditionally independent and conditionally identically distributed given X.

1. Show $I(X;Y_1,Y_2) = 2I(X;Y_1) - I(Y_1;Y_2)$.
2. Conclude that the capacity of the channel



is less than twice the capacity of the channel



Problem 9.4 (The two-look Gaussian channel.)

Consider the ordinary Shannon Gaussian channel with two correlated looks at X, i.e., $Y = (Y_1, Y_2)$, where

$$\begin{aligned} Y_1 &= X + Z_1 \\ Y_2 &= X + Z_2 \end{aligned}$$

with a power constraint P on X, and $(Z_1, Z_2) \sim \mathcal{N}_2(\mathbf{0}, K)$, where

$$K = \begin{bmatrix} N & N\rho \\ N\rho & N \end{bmatrix}.$$

Find the capacity for

1. $\rho = 1$,
2. $\rho = -1$,
3. $\rho = 0$.