AP Statistics (Cookbook Summary)

Statistical Inference
Stat 419/519
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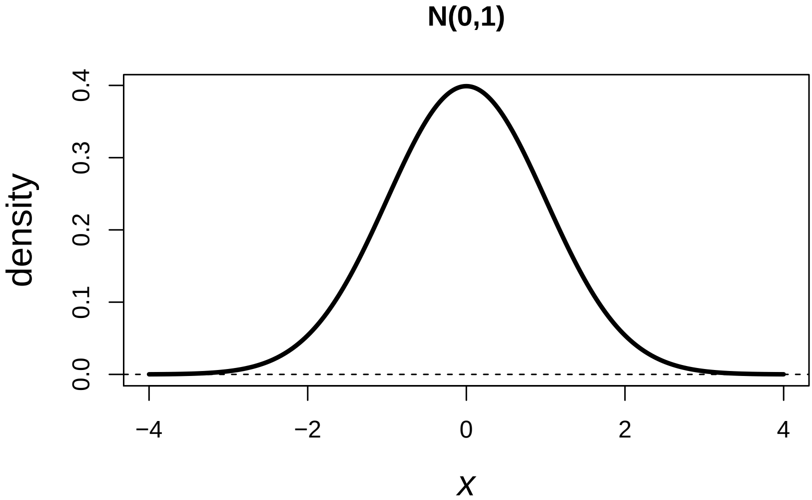
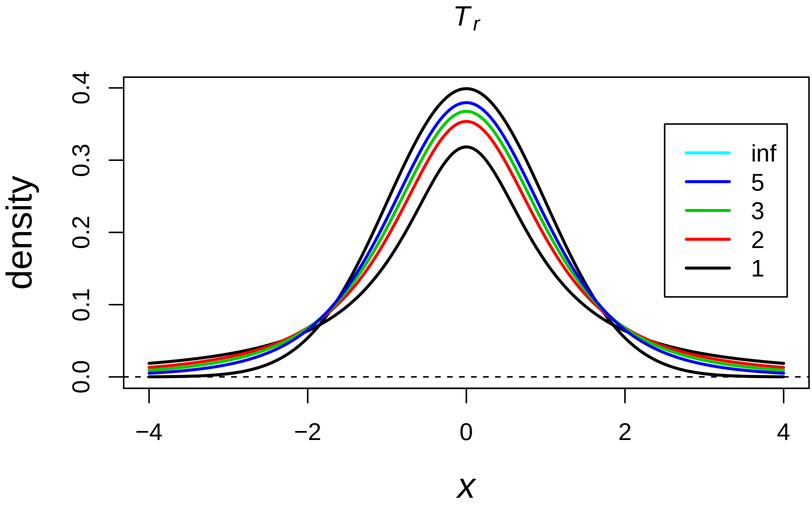
Statistical Tests

- In this course, we will develop a theoretical framework for statistical inference and tests.
- Although our favorite high-school level AP Statistics course is not a prerequisite, it is useful to know the formulae presented there.
- In fact, we will not have time to derive each of these!
- We begin by listing the 4 most important sampling distributions.
- All statistical tests basically attempt to characterize when the data are too far away from the assumed hypothesis.

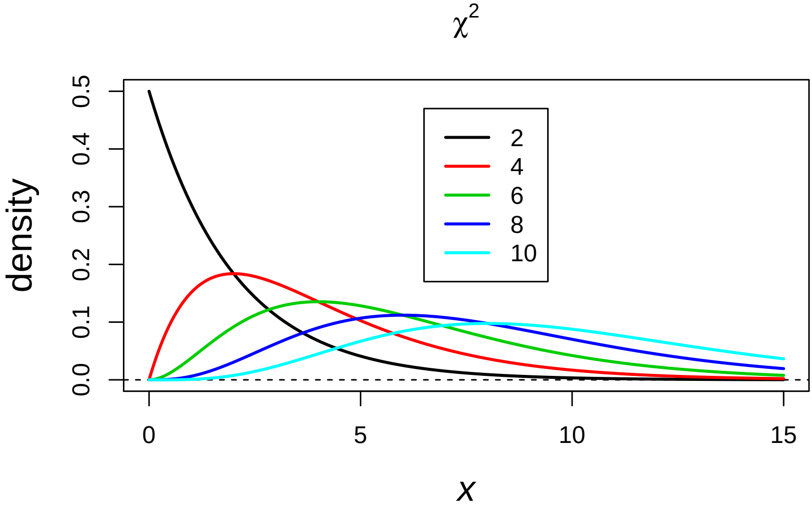
Sampling Distributions: Normal



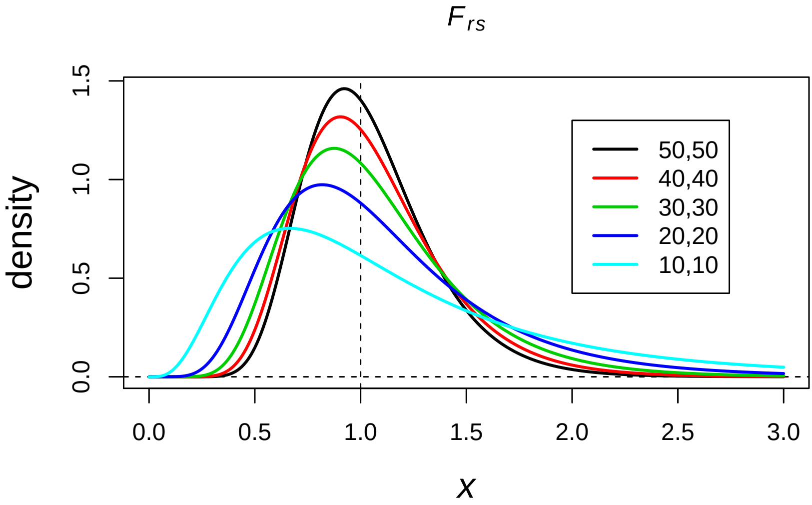
Sampling Distributions: Student's T_r



Sampling Distributions: Chi-Squared χ^2_r



Sampling Distributions: Snedecor's $F_{r,s}$



Most Important Tests: One-Sample T-test

To test the null hypothesis

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0 \, ,$$

the test statistic is

$$T_{n-1} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

where

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$
$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \, .$$

Two-Sample T-test

To test the null hypothesis

$$H_0 : \mu_x = \mu_y$$
$$H_1 : \mu_x \neq \mu_y \,,$$

the test statistic is

$$T_{n-2} = \frac{\bar{X} - \bar{Y}}{S_P \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}}$$

where

$$S_P^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}, .$$

F-test for Equality of Variances

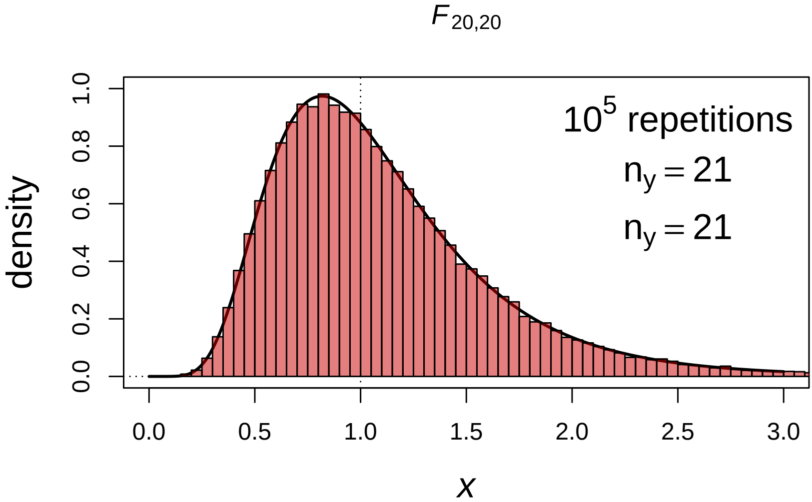
To test the null hypothesis

$$H_0 : \sigma_x^2 = \sigma_y^2$$
$$H_1 : \sigma_x^2 \neq \sigma_y^2 \, ,$$

the test statistic is

$$F_{n_x - 1, n_y - 1} = \frac{S_x^2}{S_y^2} \, .$$

F-test Simulation (Check)



χ^2-tests for Goodness-of-Fit

To test the null hypothesis

$$
\begin{aligned}
H_0 &: \text{ model gives predictions } e_1, e_2, \ldots, e_k \\
H_1 &: \text{ predictions not close to observed counts } o_1, o_2, \ldots, o_k \,,
\end{aligned}
$$

the test statistic is either

$$\sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i} \qquad \text{or} \qquad \sum_{i=1}^{r} \sum_{j=1}^{s} \frac{(o_{ij} - e_{ij})^2}{e_{ij}} \,,$$

which is approximately χ^2_{df} . The number of degrees of freedom (df) depends on which of the 3 types of models are under consideration: goodness-of-fit, contingency table, or multinomial.

One-Way ANOVA F-Test

To test the null hypothesis

$$
\begin{aligned}
H_0 &: \ \mu_1 = \mu_2 = \cdots = \mu_k \\
H_1 &: \ \text{the } k \text{ means are not all equal}\,,
\end{aligned}
$$

the test statistic is

$$
F_{k,n-k} = \frac{MS_{treatment}}{MS_{error}}\,.
$$

T-Test for Correlation Coefficient

To test the null hypothesis

$$H_0: \ \ \rho = 0$$
$$H_1: \ \ \rho \neq 0\,,$$

the test statistic is

$$T_{n-2} = R\sqrt{\frac{n-2}{1-R^2}}\,,$$

where

$$R = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2} \cdot \sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}\,.$$

T-Test for Linear Regression Coefficient

To test the null hypothesis

$$H_0 : \ \beta = \beta_0$$
$$H_1 : \ \beta \neq \beta_0 ,$$

the test statistic is

$$T_{n-2} = \frac{\hat{\beta} - \beta_0}{SE_{\hat{\beta}}} .$$