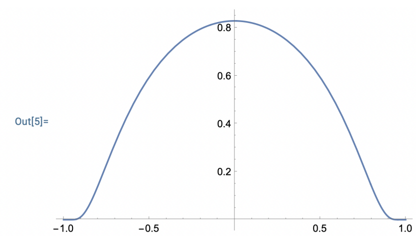
Solution Homework 5 Stat 550 (11-25-23)

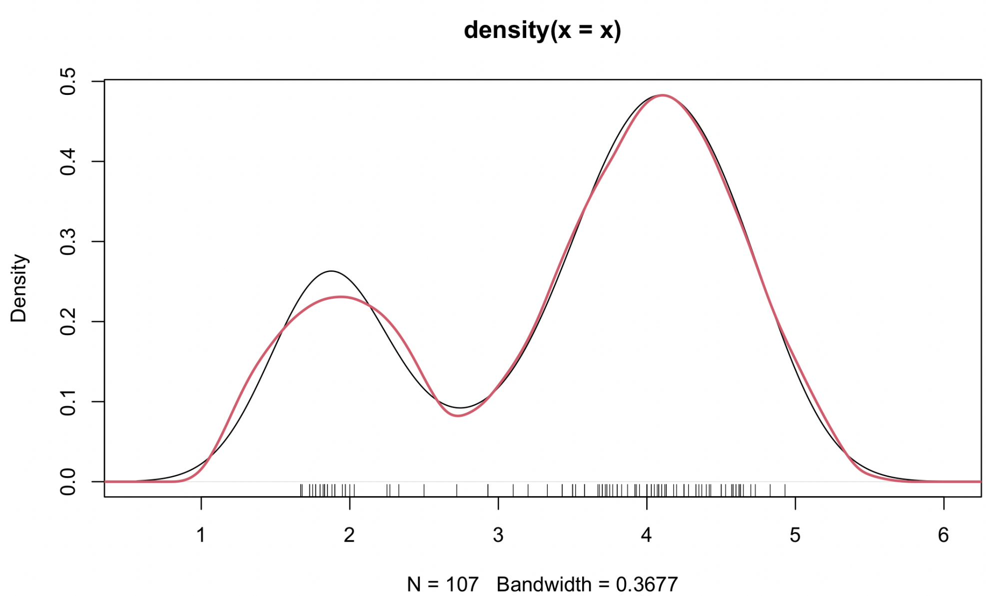
1. Using kernel in Problem 6-25.

   a. Mathematica code:

```
tmp = Exp[-1/(1-t**2)]
con = NIntegrate[ tmp,t,-1,1] (* 0.443994 *)
kt = tmp/con (* normalize kernel *)
vr = NIntegrate[ t**2 kt, t,-1,1 ] (* variance = 0.158114 *)
h$equiv = .3677 / Sqrt[vr] (* 0.924717 *)
```

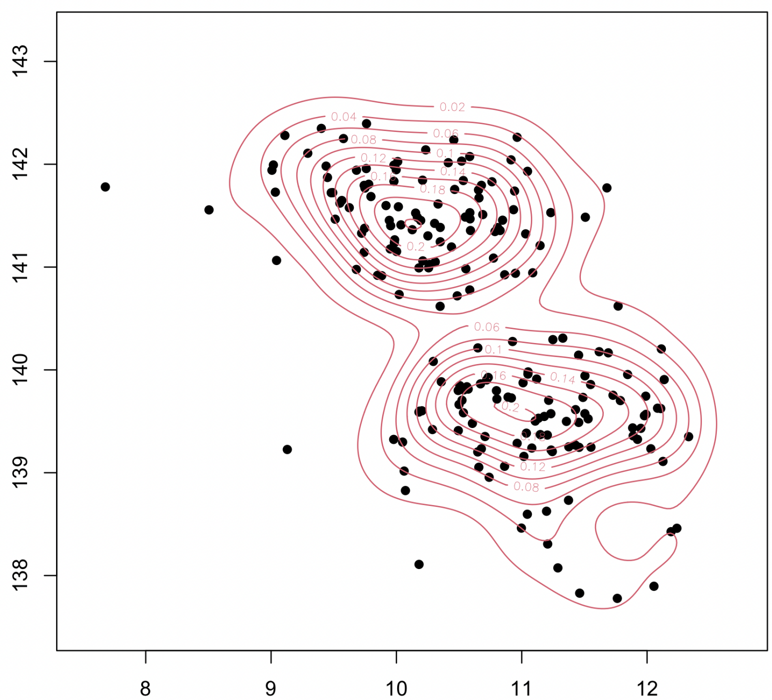


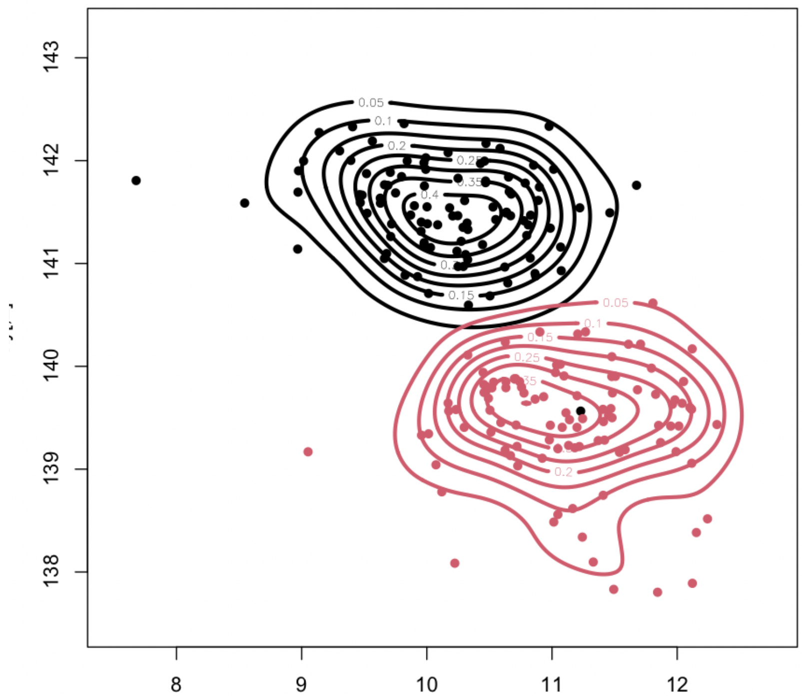
   b. Compare normal and this kernel using the geyser data. Note $h = 0.3677$ from the x-label of the density function plot.



```
kern = function(x,tk,h) { n = length(x); nt = length(tk)

  kt = function(xi,tk,h) { ti = abs( (xi-tk)/h ); iti = seq(nt)[ti<1]
 fxi = rep(0,nt); fxi[iti] = 2.25228 * exp( -1/(1-ti[iti]**2) ) / h
        return(kt) }

 ftk = rep(0,nt)
    for( i in 1:n )  ftk = ftk + kt(x[i],tk,h) ; ftk = ftk/n
        return(ftk) }

   x = dget("geyser.dat")
    plot( density(x) ); rug(x)
 kde = kern(x,tk,h=0.924717)
        lines(tk,kde,col=2,lwd=2)
```

2. Source the R code in the file `ash.q`:




143
142
141
140
139
138
8
9
10
11
12
0.05
0.1
0.2
0.25
0.35
0.4
0.15
0.05
0.1
0.15
0.25
0.35
0.2

3. Problem 7-2. Peter Hall's identity.

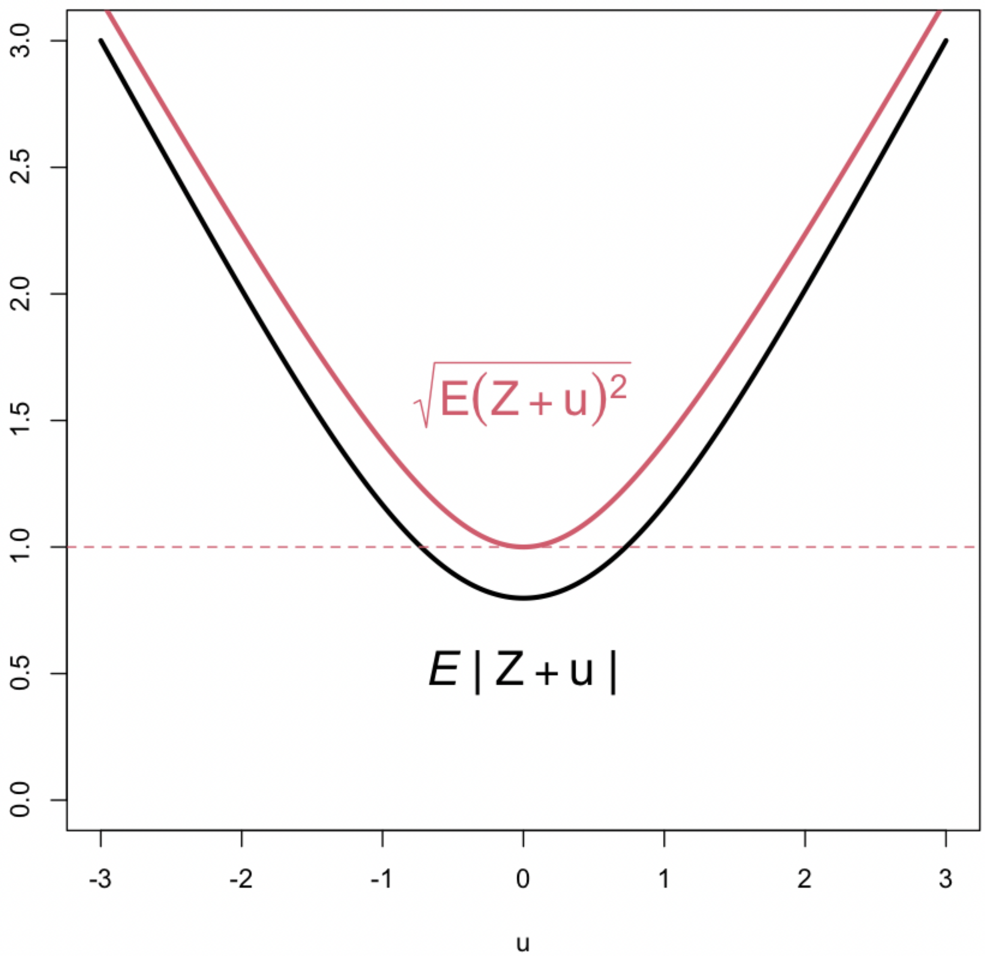
$$
\begin{aligned}
E|Z+u| &= \int_{-\infty}^{\infty} |z+u| \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \\
&= \int_{-\infty}^{-u} -(z+u) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + \int_{-u}^{\infty} (z+u) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \\
&= \int_{-\infty}^{-u} -z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz - u \int_{-\infty}^{-u} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + \\
&\quad \int_{-u}^{\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + u \int_{-u}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \\
&= \left[\int_{-\infty}^{-u} + \int_{-u}^{\infty} - \int_{-u}^{\infty} \right] \left(-z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \right) - u\Phi(-u) + \\
&\quad \int_{-u}^{\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + u \left[1 - \Phi(-u) \right] \\
&= -\int_{-\infty}^{\infty} z\phi(z) + \int_{-u}^{\infty} z\phi(z) - u\Phi(-u) + \int_{-u}^{\infty} z\phi(z) + u - u\Phi(-u) \\
&= 0 + 2\int_{-u}^{\infty} z\phi(z) - 2u\Phi(-u) + u \,.
\end{aligned}
$$

The first integral may be computed by the change of variables $v = z^2/2$,

$$
\begin{aligned}
2\int_{-u}^{\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz &= 2 \frac{1}{\sqrt{2\pi}} \int_{u^2/2}^{\infty} e^{-v} dv \\
&= 2 \frac{1}{\sqrt{2\pi}} \left[-e^{-v} \Big|_{u^2/2}^{\infty} \right] \\
&= -2 \frac{1}{\sqrt{2\pi}} \left[e^{-\infty} - e^{-u^2/2} \right] \\
&= 2\phi(u) \,.
\end{aligned}
$$

Finally, $\Phi(-u) = 1 - \Phi(u)$, so that we have

$$
\begin{aligned}
E|Z+u| &= 2\phi(u) - 2u[1 - \Phi(u)] + u \\
&= 2\phi(u) + 2u\Phi(u) - u \,.
\end{aligned}
$$



Comparing the absolute error function to the root-squared-error function. Not surprising that the optimal bandwidths for both are very similar.