1. When might you plot the n points $\big(\log(x_i),\ \log(y_i)\big)$ on a scatter diagram rather than the n raw data points $\{(x_i, y_i)\,,\ i = 1, 2, \ldots, n\}$?

2. For the each of the following seven measurements, indicate whether it falls on the **nominal, ordinal, interval, or ratio scale**. Make a comment if you feel it is not 100% obvious, but not required.

   - Hurricane category (1-5)
   - Difference in height of mother and daughter
   - Absolute difference in their heights
   - Primary color of Olympic team uniform
   - Average score of 9 judges in skating competition
   - Hang time for triple Lutz jump for ice skater
   - Temperature in competition hall (°C)

3. Suppose X_1 and X_2 are independent measurements from our spinner, i.e., $U(0, 1)$. Their domain is the unit square $(0, 1) \times (0, 1)$. For each event given below, sketch the event in the unit square and compute its probability. *Hint: Use geometric probability.*

   (a) $Prob\left(X_1 + X_2 \geq \frac{3}{4}\right)$ (b) $Prob\left(\min(X_1, X_2) \geq \frac{1}{3}\right)$

   (c) $Prob\left(X_1 < \frac{3}{10}\right)$ (d) $Prob\left(\left(X_1 - \tfrac{1}{2}\right)^2 + \left(X_2 - \tfrac{1}{2}\right)^2 \geq \frac{1}{4^2}\right)$

4. Knowing only Kolmogorov's axioms, for example $P(A \uplus B) = P(A) + P(B)$, show that these two general formulae hold:

   (a) $P(A \cup B) = P(A) + P(B) - P(AB)$.

   (b) $P(A|B) + P(A^*|B) = 1$. *Hint: Use also the definition of conditional probability.*

5. Suppose the following 3 probabilities are true:

   $$P(A) = \frac{4}{10} \qquad P(B) = \frac{3}{10} \qquad P\big((A \cup B)^*\big) = \frac{42}{100}\,.$$

   Are events A and B independent?

6. What is the probability of rolling a total of 5 (pips) on a single toss of 3 fair six-sided dice?

7. Count the complete list of **all events** associated with the toss of a single six-sided die:

   (a) Supposing the die is fair? *Hint: All simple and compound events? You need not list them, just derive the total number.*

   (b) Supposing the die is "loaded" (i.e. not fair, but all 6 simple outcomes still have nonzero probabilities adding to 100%), does your answer change?

8. In five-card draw Poker, what is the probability of getting exactly **one pair**?

9. Suppose the random variable X has density

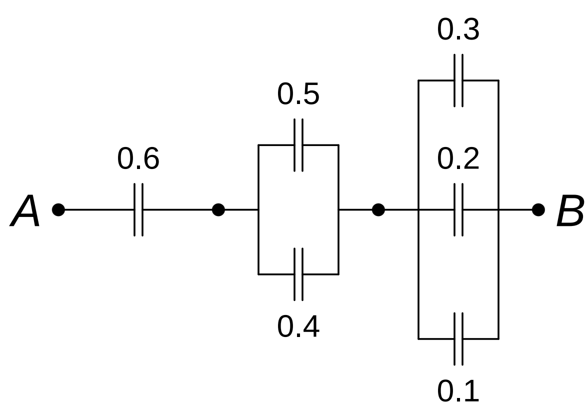
$$f(x) = \begin{cases} k \cdot x & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

   (a) What is k?
   (b) What are the first two non central moments of X? Hint: You may use direct integration rather than fussing with the MGF.
   (c) What is μ using your answer from part (b)?
   (d) What is σ^2 using your answer from part (b)?

10. In a sequence of independent Bernoulli trials $B(1, p)$, let the random variable X be the "time" (sequence number) when the **second success** first occurs.
   (a) Derive the PDF of X; i.e., give the precise formula for $f(x)$.
   (b) Show that $f(x)$ is a valid PDF.

11. A student has mastered 80% of the material. Assume this means the student has an 80% chance of getting full credit (10 points) on each problem, and all problems are independent of the other problems. But if the student does not know the answer, the student guesses at one of the four multiple choice options. Notice there is no partial credit on this exam, each question scores either a 10 or a 0.
   (a) What is the probability the student correctly answers a question?
   Hint: Partition on an relevant event.
   (b) If there are ten problems, what is the student's expected score?
   Hint: What is the PDF of the number of correct answers on the exam?
   (c) Write down the formula for the probability the student scores at least a 90 of 100 on the exam?

12. Suppose the 6 components in the diagram below are all stochastically independent. The probability each works is indicated on the diagram. What is the probability that a signal will reach B when sent from A?



13. If events A and B are on a complete list of events, then so should the events A^*, B^*, and $A \cap B$. From just the knowledge of these three facts (and any appropriate general set operations), show that $A \cup B$ will be on the list as well.