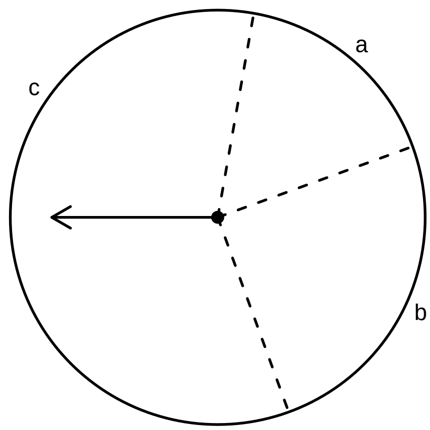
1. List all events associated with this spinner, whether the spinner arrow lands in arc a, b, or c around the circle? (Remark: In the figure below, the spinner landed in the arc labelled "c.")



2. For the each of the following five measurements, indicate whether it falls on the **nominal, ordinal, interval, or ratio scale**. Make a comment if you feel it is not 100% obvious.

   - The time for a college team to complete the beer-bike race
   - The name of the college you are a member of at Rice
   - A grade on the 1st exam
   - Temperature in degrees Celcius
   - The Richter scale for measuring the severity of an earthquake

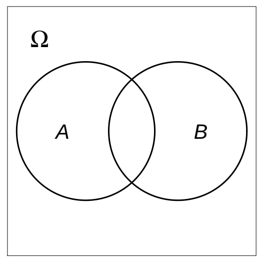
3. Demonstrate that the set of all integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is countable.

4. The probability that either events A, B, or both occur is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), .$$

Suppose we are interested in the probability that exactly *one* of the events A and B will occur (i.e., not both).

(a) Write this event using set theory. Call this new event C.



(b) Derive a formula for the event C using $P(A)$, $P(B)$, and $P(A \cap B)$.
Hint: Don't just write down (guess) the answer. Instead, recall that

$$A = AB^c \uplus AB \qquad \text{and} \qquad B = AB \uplus A^c B\,.$$

5. Do there exist events A and B satisfying

$$\begin{array}{lr} (1) & P(A) = 0.7 \\ (2) & P(B) = 0.6 \\ (3) & P(A \cap B) = 0.2\,? \end{array}$$

Why or why not?

6. When discussing Kolmogorov's Axioms, we try to use as few assumptions as possible. For example, when describing the list of all events, $\mathcal{F}$, we say

(1) if $A \in \mathcal{F}$, then so is A^c, its complement

(2) if A and B are events in $\mathcal{F}$, then so is $A \cap B$, their intersection.

Show by using elementary set operations that the event $A \cup B$ is also in $\mathcal{F}$.

Hint: Draw a Venn Diagram such as in problem 4 and highlight the set $A^c \cap B^c$.

7. You are teaching your niece how to play poker. Rather than using a full deck of 52 cards, you choose only the single digit cards $2-9$ **and** only spades and hearts. You deal 3 (rather than 5) cards for a poker hand.
   (a) How many equally-likely poker hands are there?
   (b) For each of the "good hands," what is the count?

8. Roll a pair of 4-sided dice (as in the homework) and record the results as (X, Y). Define the derived random variable
$$V = \max(X, Y)\,.$$
   What is the PMF of V?

9. Suppose the random variable X has density
$$f(x) = \begin{cases} \frac{1}{2} \cdot x & \text{for } 0 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$
   (a) Show $f(x)$ is a PDF.
   (b) Compute (directly) the first 4 non-central moments. (Do not use the MGF.)
   (c) What are the mean, variance, standard deviation, skewness, and kurtosis of X? (Write down the formulae. Evaluate if you have time.)

10. Suppose you have been trained in CPR (cardiopulmonary resuscitation) and that you

live at the middle of a long, straight hall. We label the hall as the interval $(-100, 100)$, i.e., you are at 0. Emergencies occur randomly in the hall. Let the random variable X measure the distance between you and a random emergency.

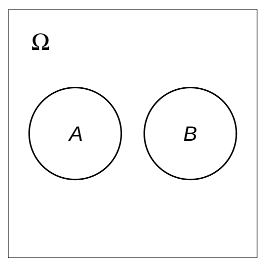
(a) What is the CDF of X? That is, what is the probability an emergency occurs at a distance less than or equal to x from you? Sketch your answer.

(b) Derive the PDF of X; that is, give the precise formula for $f(x)$. Sketch it.

(c) What is the mean of X, that is, compute $\mu = E[X]$?

11. Consider 2 events A and B with special assumptions.

(a) First, suppose A and B look like



What is $P(A|B) + P(A|B^c)$ for this assumption?

(b) Next, suppose A and B are independent. What is $P(A|B) + P(A|B^c)$ now?

(c) Does part (b) contradict part (a)?

12. Suppose events A and B are *not* independent and that, in fact, satisfy

$$P(A|B) > P(A) .$$

One of the following is true:

(a) $P(B|A) = P(B)$
(b) $P(B|A) < P(B)$
(c) $P(B|A) > P(B)$
(d) cannot tell, in general.

Find it and show that is it true. *Hint: Use Bayes formula.*

13. A newlywed couple decides they wish to have a family with (at least) 2 boys and girls. They plan to stop when that happens. (For example, if they currently have 5 boys and 1 girl, they will try again until they have a second girl, then stop.) Suppose $p = 1/2$ for a girl or boy baby.
    (a) What is the probability their family will have exactly 4 children?
    (b) What is the probability their family will have exactly 5 children?