1. An urn contains 4 identical balls labeled with one of the letters {a}, {b}, {c}, and {d} (one letter on each ball). List all events associated with a single random draw of a ball from the urn.

2. For the each of the following five measurements, indicate whether it falls on the **nominal, ordinal, interval, or ratio scale**. Make a short comment justifying your choice (and a longer comment if you feel it is not 100% obvious).

   - The time spent studying for this exam
   - The name of the university your best high school friend attends
   - The total of all your points at the end of this course
   - Temperature in degrees Kelvin
   - The strength of a tornado measured on the Fujita scale: F0, F1, F2, F3, F4, or F5

3. Demonstrate that the set of all rational numbers between -1 and 1 is countable.

4. For the events given below defined on the unit square with the location of the point (X, Y) following a uniform distribution, sketch the event and compute its probability. In each case, the event A is the set of points (X, Y) satisfying the constraint

   (a) $Y > \frac{4}{3} - X$

   (b) $\min(X - 0.1, Y) > 0.6$

   (c) $(X - 0.3)^2 + (Y - 0.6)^2 \leq \frac{1}{64}$

5. When discussing Kolmogorov's Axioms, we try to use as few assumptions as possible. Suppose, for example, when describing the list of all events, $\mathcal{F}$, we start with only the 2 assumptions:

   (1) if $A \in \mathcal{F}$, then so is A^c, its complement

   (2) if A and B are events in $\mathcal{F}$, then so is $A \cup B$, their union.

   Show by using elementary set operations that the event $A \cap B$ is also in $\mathcal{F}$.

   Hint: Draw a Venn Diagram and highlight the set $A^c \cup B^c$.

6. Questions about events A, B and C.

(a) Suppose event B is independent of event A, that is, $P(B|A) = P(B)$. Show that event A must be independent of event B.

(b) Suppose events A and B are *not* independent and that B is more likely to occur when A does, that is,

$$P(B|A) > P(B)\,.$$

Now A cannot be independent of B. One of the following is true:

$$\begin{array}{l}(a)\ P(A|B) = P(A)\\(b)\ P(A|B) < P(A)\\(c)\ P(A|B) > P(A)\\(d)\ \text{cannot tell, in general.}\end{array}$$

Find it and show that is it true.

(c) Show the following is always true.

$$P(A|BC) + P(A^c|BC) = 1\,.$$

Hint: You may assume all relevant events have non-zero probability.

7. Suppose the random variable X has density

$$f(x) = \begin{cases} c \cdot (1 - x^2) & \text{for } -1 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find the value of the constant c so that $f(x)$ is a PDF.

(b) Compute (directly) the mean and standard deviation. (Do not use the MGF approach.)

8. While playing poker with a well-shuffled full deck of 52 cards, the first three (of five) cards that you are dealt are the set (event)

$$\{\ 4\spadesuit\,,\ 7\heartsuit\,,\ 7\clubsuit\ \}\,.$$

(a) How many equally-likely ways are there to complete your 5 cards?

(b) How many ways are there of getting 4-of-a-kind or one pair?

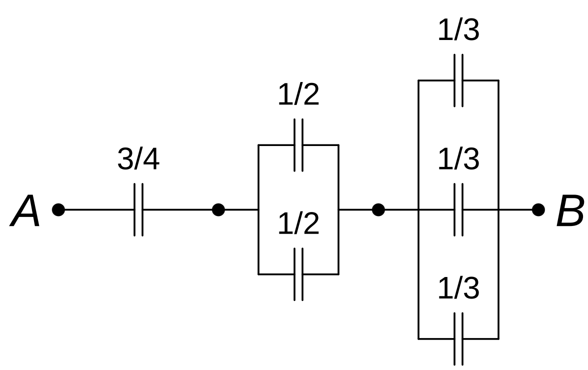
(c) List all the remaining possible hands you may end up with, and how many for each.

Poker Hands (in order of quality):

1. Royal Flush (AKQJ10, one suit)
2. Other Straight Flushes (45678, one suit, Ace low OK in Texas Hold'em)
3. Four of a Kind (KKKK8)
4. Full House (AAA22, three-of-a-kind plus one pair)
5. Flush (5 cards same suit, not in sequence)
6. Straight (23456 different suits, Ace low OK in Texas Hold'em)
7. Three of a Kind (555JK)
8. Two pair (AAQQ7)
9. Pair (AA268)
10. "Nothing" (everything else — Jack high, for example)

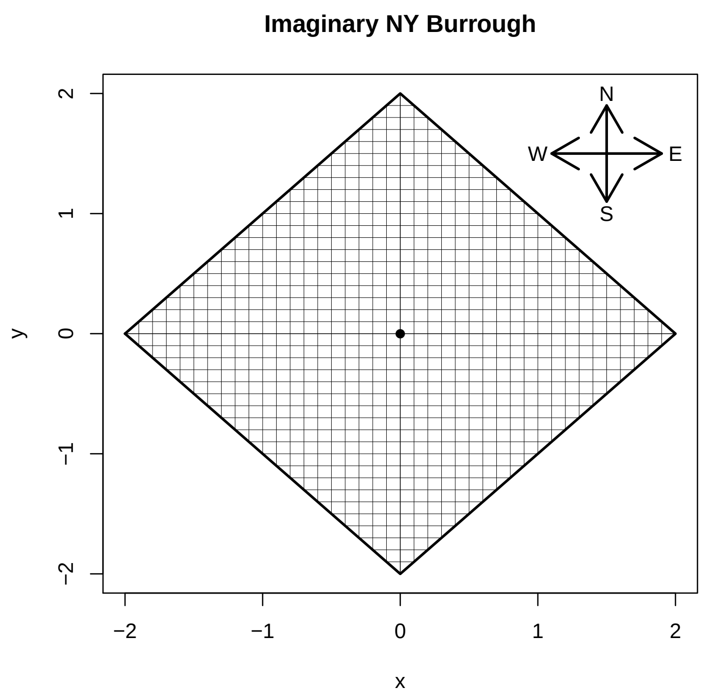
Order of suits (best to worst): *Spades, Hearts, Diamonds, Clubs.*

9. Suppose the 6 components in the diagram below are all stochastically independent. The probability each works is indicated on the diagram. What is the probability that a signal will reach B when sent from A?



10. Suppose you are an urban planner for a new burrough with the geometry shown in the figure (a square rotated 45 degrees). Some of the grid of streets and avenues are also shown. You are tasked with placing 4 hospitals to serve the region, supposing demand will be uniformly distributed over the region.

    (a) Where should you locate them so all 4 regions are "fair" to all the residents? Indicate on the figure what region each hospital should serve.



(b) How is this related to one of our homework problems? For example, what is the furtherest anyone will be from a hospital?

(c) In particular, if the random variable D measures the distance from a random emergency in the burrough to the nearest of the 4 hospitals, what is the CDF of D, that is, what is $F(d) = Prob(D \leq d)$?